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Accuracy Evaluation of Boundary Interface in Sound Field Simulation Using High-order FDTD Methods.

高次差分を用いた音場 FDTD 数値解析における媒質間境界の精度評価

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1. Introduction

Numerical analysis of acoustic fields has become investigated widely as a result of recent computational progress. FD-TD (finite difference time domain) method is very widely used for time domain numerical analysis. Many numerical analysis of sound propagation by the FD-TD method have been reported for a few decades.

In acoustic numerical analysis using FD-TD method, generally, the analysis region is not a uniform medium. Therefore, in the FD-TD analysis, the treatment of the boundary interface between media is important. Calculation need to satisfy the boundary condition of the sound field[1]. Generally, FDTD method uses the orthogonal grid, and discretizes the analytical domain. Each discretized grid is set the medium constant (Generally, density and bulk modulus). The boundary interface exists not only on the grid but also between the grids. The handling of medium constant is required to pay attention.

In this study, we examined the setting of the medium constant for boundary interface in the 4th- and 6th- order FDTD methods, and evaluated the accuracy.

2. FDTD method

We present formulation of FDTD method. For simplicity, let us examine a one-dimensional (1D) model. Governing equations in the acoustic field show eqs.(1) and (2).

$$\rho \frac{\partial}{\partial t}\vec{v} = -\frac{\partial}{\partial x} p \qquad (1)$$

$$\frac{\partial}{\partial t} p = -K \frac{\partial}{\partial x}\vec{v} \qquad (2)$$

In these equations, ρ denotes the density of the medium, K is the bulk modulus, ρ is the sound pressure, $\vec{v}$ is the particle velocity.

By the applying FD-TD algorithm, eqs. (3) and (4) are obtained using discretized components of sound pressure and particle velocity on grid points:

$$\begin{aligned} p\big|_i^{n+1} = p\big|_i^{n} &- A\dot{K}\big|_i \Delta t \frac{v_x\big|_{i+1/2}^{n+1/2} - v_x\big|_{i-1/2}^{n+1/2}}{\Delta x} \\ &- B\ddot{K}\big|_i \Delta t \frac{v_x\big|_{i+3/2}^{n+1/2} - v_x\big|_{i-3/2}^{n+1/2}}{3\Delta x} \\ &- C\dddot{K}\big|_i \Delta t \frac{v_x\big|_{i+5/2}^{n+1/2} - v_x\big|_{i-5/2}^{n+1/2}}{5\Delta x} \end{aligned} \qquad (3)$$

$$\begin{aligned} v_x\big|_{i+1/2}^{n+1/2} = v_x\big|_{i+1/2}^{n-1/2} &- A \frac{\Delta t}{\dot{\rho}\big|_{i+1/2}} \frac{p\big|_{i+1}^{n} - p\big|_{i}^{n}}{\Delta x} \\ &- B \frac{\Delta t}{\ddot{\rho}\big|_{i+1/2}} \frac{p\big|_{i+2}^{n} - p\big|_{i-1}^{n}}{3\Delta x} \\ &- C \frac{\Delta t}{\dddot{\rho}\big|_{i+1/2}} \frac{p\big|_{i+3}^{n} - p\big|_{i-2}^{n}}{5\Delta x} \end{aligned} \qquad (4)$$

Here, we assume that the calculation is for a lossless medium. Δx and Δt respectively denote the grid size and the time step. i represents the spatial discrete point correspond to x-coordinate, and n is discrete time.

In these equations, FDTD(2,6) method uses A=15/128, B=-25/128, C=3/128, whereas FDTD(2,4) method uses A=9/8, B=-1/8, where, A+B+C=1 in FDTD(2,6) ,and A+B=1 in FDTD(2,4) method.

3. Treatment of boundary interface in FDTD methods

In the treatment of the boundary interface, it is necessary to satisfy the Dirichlet and Neumann boundary condition[2]. According to these conditions, we derive the equivalent constant for the bulk modulus and density on the boundary interface between medium 1 (K_1, ρ_1) and medium 2 (K_2, ρ_2). These constant is shown below:

$$K = \frac{(l_1 + l_2)K_1 K_2}{l_2 K_1 + l_1 K_2} \qquad (5)$$

$$\rho = \frac{l_1 \rho_1 + l_2 \rho_2}{l_1 + l_2} \qquad (6)$$

where l_1 and l_2 are the length of medium1 and medium2[1,2].

For example, in FDTD(2,4) method, we

show the equivalent constants when the boundary interface is located at $x=(i_0+\frac{1}{5})\Delta x$ (See Fig.1).

The bulk modulus constant:

$$\dot{K}\big|_{i_0-1}=K_1,\quad \ddot{K}\big|_{i_0-1}=\frac{30K_1K_2}{3K_1+27K_2} \tag{7}$$

$$\dot{K}\big|_{i_0}=\frac{10K_1K_2}{3K_1+7K_2},\quad \ddot{K}\big|_{i_0}=\frac{30K_1K_2}{13K_1+17K_2} \tag{8}$$

$$\dot{K}\big|_{i_0+1}=K_2,\quad \ddot{K}\big|_{i_0+1}=\frac{30K_1K_2}{23K_1+7K_2} \tag{9}$$

The density constant:

$$\dot{\rho}\big|_{i_0-\frac{1}{2}}=\rho_1,\quad \ddot{\rho}\big|_{i_0-\frac{1}{2}}=\frac{22\rho_1+8\rho_2}{30} \tag{10}$$

$$\dot{\rho}\big|_{i_0+\frac{1}{2}}=\frac{2\rho_1+8\rho_2}{10},\quad \ddot{\rho}\big|_{i_0+\frac{1}{2}}=\frac{12\rho_1+18\rho_2}{30} \tag{11}$$

$$\dot{\rho}\big|_{i_0+\frac{3}{2}}=\rho_2,\quad \ddot{\rho}\big|_{i_0+\frac{3}{2}}=\frac{2\rho_1+28\rho_2}{30} \tag{12}$$

By same treatment, the medium constants can be set in FDTD, FDTD(2,6) and more high-order methods.

4. Result and discussion

We show the numerical results obtained using above the medium constants. In calculation model the two medium is considered; medium1(K_1,ρ_1) and medium2(K_2,ρ_2) in 1-D analysis. The boundary interface is assumed to be located at $x=i_0\Delta x$, $x=(i_0+1/5)\Delta x$, $x=(i_0+2/5)\Delta x$, $x=(i_0+3/5)\Delta x$, $x=(i_0+4/5)\Delta x$, and $x=(i_0+1)\Delta x$.

Calculation parameters are :i_0 = 35 m; grid size Δx =0.05 m; number of grid points Nx = 2000, ρ_1 =1.21 kg/m^3; K_1=1.4236×10⁵ N/m^2; $\rho_2=\rho_1$ [kg/m^3]; K_2=9×K_1 [N/m^2]. The initial pressure at t=0 is given as $p=e^{-\alpha(x-x_0)^2}$ [N/m^2]. In this equation, $\alpha=1/100$, x_0=25. Fig.2 shows the 1-D sound pressure distribution obtained using the FDTD (2,4) method with above treatment.

Next, we observed reflected wave from each boundary interface at x=30m. Fig.3 shows the waveforms of reflected signals.

It is confirmed that the above treatment can give valid calculation results.

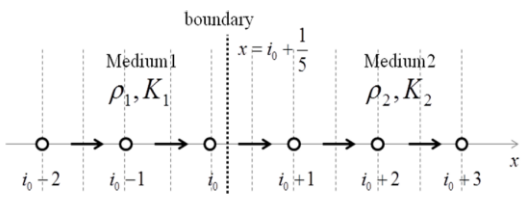

Fig.1 the model set up the boundary at $x=i_0+\frac{1}{5}$

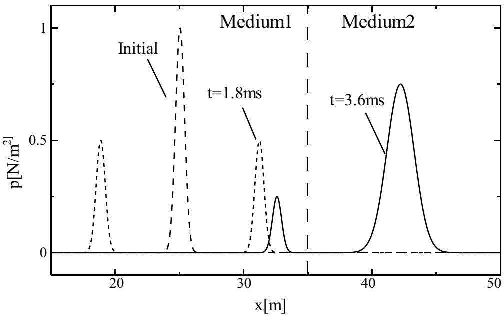

Fig.2 Distribution of the sound pressure (t=1.8ms, t=3.6ms)

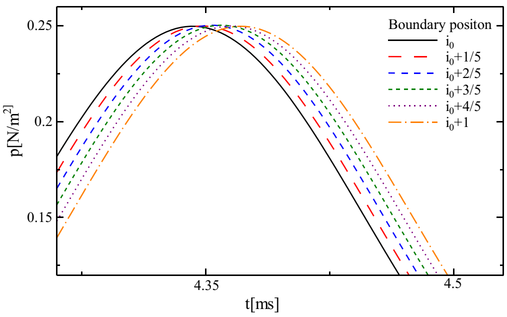

Fig.3 Distribution of the sound pressure (x=30m)

5. Conclusion

In this study, we assessed the accuracy of boundary interface in sound field simulation using high-order FDTD methods. We determined medium constants considering the length of medium. This treatment of boundary interface yields valid calculation results when the boundary interface is located between grid points

The proposed treatment is actually given by very simple procedure. Moreover, we can apply it to the higher-order FDTD methods.

References

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