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Behavior of Dielectric Mode in Complex Series Dynamics for Electromechanical Coupling Systems

電気機械結合系における複素級数力学の誘電モードの振舞い

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1. Introduction

The distributed-parameter-based treatment of vibration analysis has been developed without using any lumped parameter elements,[1-10] in which the energy mode propagates and interferes with each other in a manner of probabilistic superposition. The concept of energy mode is derived from Dirac's "complex dynamical variable" [11], in which its magnitude is proportional to the square root of stored energy.

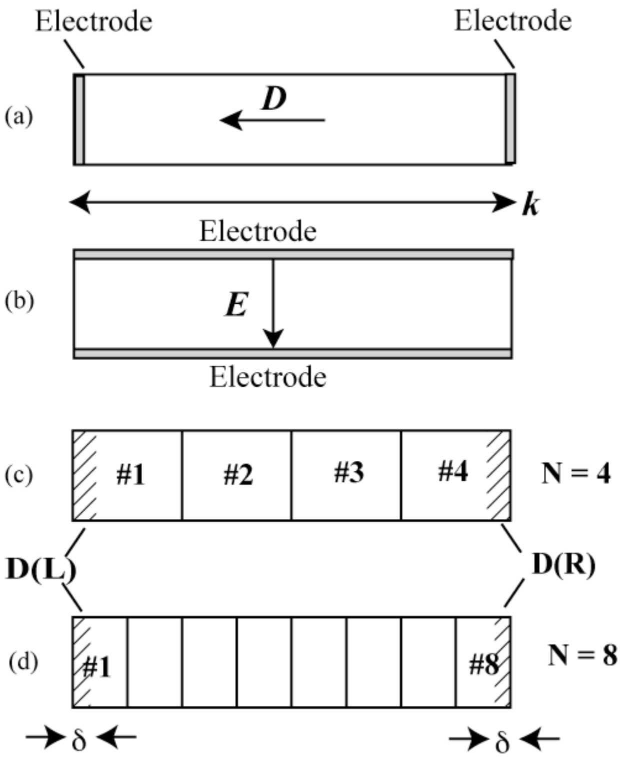

Fig. 1 Electromechanical coupling system in which the drive field spreads over the whole domain of a transducer ("whole drive"), in the cases of (a) longitudinal effect and (b) transverse effect.
(c), (d) Division of the whole domain into *N* layers. *D*(*L*) and *D*(*R*) are spatial infinitesimal (delta-function-wise) domains (δ → 0) at the left edge and the right edge inside the drive domain, respectively, on which an energy concentration phenomenon in dielectric mode occurs, and the polarities of mode at both edges are the same.

This methodology can also deal with phenomena of mode coupling: for example, the phenomena of mechanical mode coupling by considering the interaction between two "elastic modes" [9,10] and the phenomenon of electro-mechanical coupling by considering the interaction between elastic mode and "dielectric mode",[1,4] which makes the resonance frequencies of the system shifted. In an electromechanical coupling system, elastic mode and dielectric mode are considerd to be coupled in two types of unitary (energy-conservative) processes; one is termed "continuous interaction" in which an infinitesimal

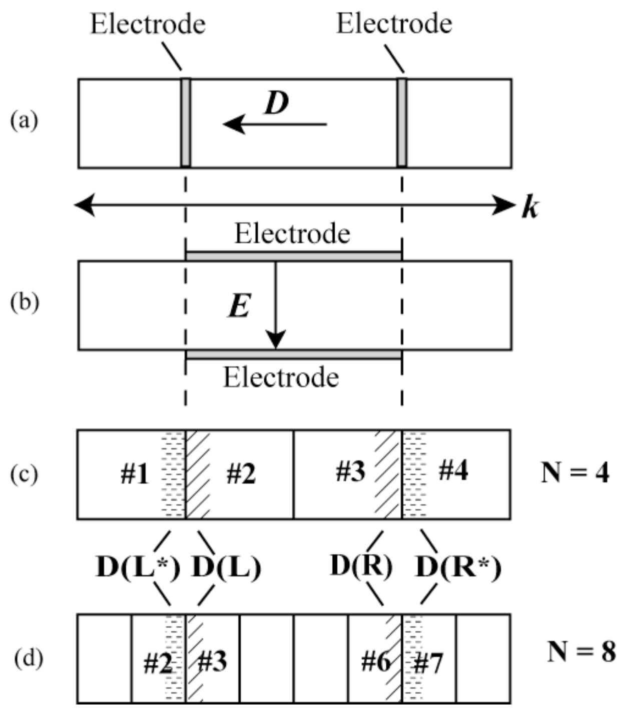

Fig. 2 Corresponding configuration to Fig. 1 in the case of "partial drive" in which the domain of layers #3 and #4 for *N* = 4 or the domain of layers #3 to #6 for *N* = 8 is driven. *D*(*L**) and *D*(*R**) are spatial infinitesimal domains outside the drive domain, neighboring on *D*(*L*) and *D*(*R*), respectively. The concentrated energy of dielectric mode "seeps" from *D*(*L*) and *D*(*R*) into *D*(*L**) and *D*(*R**), respectively, but the polarities of mode become the inverse.

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coupling between the two modes occurs in a finite interaction length in an integral manner, and the other is termed "point interaction" in which a finite coupling between the two modes occurs in an infinitesimal interaction length or spatial point.[1)] In this study, the behavior of dielectric mode is investigated for the purpose of clarifying the physical background to the above interaction processes.

2. Energy Concentration Phenomenon in Dielectric Mode

With regard to the dielectric mode alone without any unitary processes with the elastic mode, the calculation of Neumann series[3)] (the infinite geometrical series of matrices) leads to the following results.

In the case shown in **Fig. 1** (the case of "whole drive"), the dielectric mode has a tendency to concentrate at the edges on which the drive field terminates spatially (domains $D(L)$ and $D(R)$ in the figure). The volume of the domain in which the energy concentration occurs is infinitesimally small ($\delta \rightarrow 0$ in the figure), which can be confirmed by increasing the number of layers N on the occasion of calculation.

The degree of the concentration becomes larger as the Q-value of the system becomes larger. (Almost 100% when Q-value is sufficiently large.)

The "polarities" of dielectric mode at both edges are the same; that is, in Fig. 1(d),

$$\eta_1 = \eta_8 = +1, \quad \eta_2 = \eta_3 = \ldots = \eta_7 = 0$$

(η_i indicates the dielectric mode in layer #i), where normalization is performed appropriately.

On the other hand, in the case of "partial drive" as shown in **Fig. 2**, the dielectric mode also has a tendency of concentration at the domains $D(L)$ and $D(R)$, but "seeps" into the neighboring infinitesimal domains $D(L^*)$ and $D(R^*)$ with the inverse polarities, respectively. That is, the polarities of mode in $D(L^*)$, $D(L)$, $D(R)$, and $D(R^*)$ are "–" , "+", "+", and "–", respectively.

The degree of the seepage depends on the situation of the impedance matching at the boundaries. For example, in Fig. 2(c), when $Z_1 = 1$ and $Z_2 = 1$ (Z_i indicates the impedance in layer #i),

$$\eta_1 = -0.5, \quad \eta_2 = +0.5,$$

showing that fifty-percent seepage occurs with the inverse polarity; on the other hand, when $Z_1 \rightarrow 0$ while $Z_2 = 1$,

$$\eta_1 \rightarrow 0, \quad \eta_2 \rightarrow +1,$$

which means that the mode is confined inside ---- substantially the same situation as in the case of the whole drive shown in Fig. 1, except that the length of the transducer is substantially reduced half. (In other words, the length of a transducer does not influence the present result.)

The energy concentration phenomenon described above is considered to have relation to the point interaction described before. The point interaction between elastic mode and dielectric mode could occur at any spatial position. However, the point interaction is actually expected to occur only at the spatial points on which the energy concentration phenomenon in dielectric mode occurs.

References

1. M. Ohki: "Application of Complex Series Dynamics to Electromechanical Coupling System", Jpn. J. Appl. Phys., **50** (2011) 07HB05.

2. M. Ohki: "Impulse Response of Piezoelectric Transducer by Multiresolution Analysis of Energy Modes", Jpn. J. Appl. Phys., **46** (2007) 4474.

3. M. Ohki: "Probabilistic Superposition of Energy Modes for Treating 2N-Layered Mechanical Impedance Mismatch System", Jpn. J. Appl. Phys., **45** (2006) 4462.

4. M. Ohki: "Distributed-Parameter Based Treatment of Interaction Process between Elastic and Dielectric Energy in Piezoelectric Transducer", Jpn. J. Appl. Phys., **44** (2005) 8536.

5. M. Ohki: "Application of Spatial Multiresolution Analysis and Synthesis to Piezoelectric Inhomogeneous Drive Systems", Jpn. J. Appl. Phys., **44** (2005) 275.

6. M. Ohki and K. Toda: "Properties of Spatial Dipole Delta Function in Complex Series Dynamics", Jpn. J. Appl. Phys., **42** (2003) 3102.

7. M. Ohki and K. Toda: "Evaluation of Resonance Intensities and Frequencies for Piezoelectric Partial-Drive Systems Using Application of Mason's Equivalent Circuit and Complex Series Dynamics", Jpn. J. Appl. Phys., **41** (2002) 3413.

8. M. Ohki and K. Toda: "Calculation Principle of Stored Energy and Effective Power in Piezoelectric Transducer using Infinite Series of a Complex Dynamical Variable and its Finite Difference", Jpn. J. Appl. Phys., **40** (2001) 4615.

9. M. Ohki and K. Toda: "Equivalent Representation of Mode Coupling in Two-Dimensional Bulk Waves Using Unitary Matrix with 4 × 4 Elements", Jpn. J. Appl. Phys., **40** (2001) 3505.

10. M. Ohki and K. Toda: "Mathematical Treatment of Mode Coupling between Two Plane Waves Using Unitary Matrix and Neumann Series", Jpn. J. Appl. Phys., **39** (2000) 3060.

11. P. A. M. Dirac: The Principles of Quantum Mechanics (Oxford University Press, 1958), 4th ed.