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Three-dimensional sound field analysis using compact explicit FDTD method with GPU cluster system
GPU クラスタを用いた CE-FDTD 法による 3 次元音場解析

Takuto Ishii[1†], Takao Tsuchiya[1], and Kan Okubo[2] ([1]Doshisha Univ.; [2]Tokyo Metropolitan Univ.)
石井琢人 [1†]，土屋隆生 [1]，大久保寛 [2] ([1] 同志社大学, [2] 首都大学東京)

1. Introduction

For the large-scale numerical analysis of the 3D sound wave propagation, enormous computer resources are often required for the accurate analysis. In this paper, the compact explicit finite difference time domain (CE-FDTD) [1, 2] method is applied to reduce computer resources. The CE-FDTD method is one of the wave equation based FDTD (WE-FDTD) methods [3] and is a high accuracy version of the WE-FDTD methods. There are some variations of the CE-FDTD method. The most accurate scheme is the interpolated wideband (IWB) scheme in which the cut-off frequency is in agreement with Nyquist frequency. In this paper, some variations of the CE-FDTD method are implemented on the GPU cluster system [4], and the calculation performances and accuracy are evaluated for the 3D sound wave propagation.

2. CE-FDTD method

We here consider discretization of the 3D wave equation in the CE-FDTD method. There are 27 grid points in a discretized cell for the CE-FDTD method as shown in Fig. 1 (a). The grid intervals of x, y, and z direction are assumed to be all the same as Δ. The CE-FDTD method is one of the central difference method in which not only axis directions but also diagonal directions are considered. The discretized equation of the CE-FDTD method is given as

$$\begin{aligned} p_{i,j,k}^{n+1} &= d_1(p_{i+1,j,k}^n + p_{i,j+1,k}^n + p_{i,j-1,k}^n + p_{i,j,k+1}^n + p_{i,j,k-1}^n) \\ &+ d_2(p_{i+1,j+1,k}^n + p_{i+1,j-1,k}^n + p_{i+1,j,k+1}^n + p_{i+1,j,k-1}^n + p_{i,j+1,k+1}^n \\ &+ p_{i,j+1,k-1}^n + p_{i,j-1,k+1}^n + p_{i,j-1,k-1}^n + p_{i-1,j+1,k}^n + p_{i-1,j-1,k}^n \\ &+ p_{i-1,j,k+1}^n + p_{i-1,j,k-1}^n) + d_3(p_{i+1,j+1,k+1}^n + p_{i+1,j-1,k+1}^n \\ &+ p_{i+1,j+1,k-1}^n + p_{i+1,j-1,k-1}^n + p_{i-1,j+1,k+1}^n + p_{i-1,j-1,k+1}^n \\ &+ p_{i-1,j+1,k-1}^n + p_{i-1,j-1,k-1}^n) + d_4 p_{i,j,k}^n - p_{i,j,k}^{n-1} \end{aligned} \quad (1)$$

where $p^n_{i,j,k}$ represents sound pressure on the grid point $(i\Delta, j\Delta, k\Delta)$ at the time $t = n\Delta t$, and Δt is the time setp. The coeffiiicients $d_1 \sim d_4$ are given as follows

$$d_1 = \chi^2(1-4a+4b),\ d_2 = \chi^2(a-2b),$$
$$d_3 = \chi^2 b,\ d_4 = 2(1-3\chi^2+6a\chi^2-4b\chi^2) \quad (2)$$

where a and b are parameters for controlling accuracy, and $\chi = c_0\Delta t/\Delta$ is the CFL number.

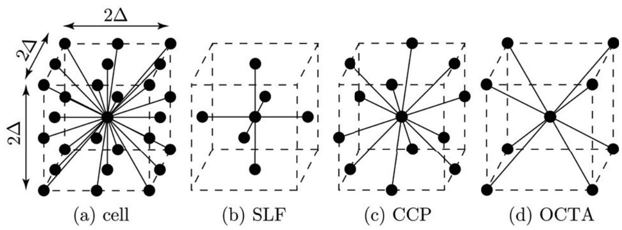

Fig. 1 A cell for CE-FDTD method.

Table 1 Numerical parameters for the various CE-FDTD schemes.

scheme	a	b	χ_m	$\bar{f}_c$
SLF	0	0	$1/\sqrt{3}$	0.196
CCP	1/4	0	1	0.333
OCTA	1/2	1/4	1	0.25
IISO	1/6	0	$\sqrt{3}/2$	0.333
IISO2	1/6	1/48	$\sqrt{3}/2$	0.333
IWB	1/4	1/16	1	0.5

In this equation, d_1 corresponds to the stencil along the axis directions as shown in Fig.1 (b), which is called standard leapfrog (SLF), (b) d_2 to the side-diagonal directions called cubic close packed (CCP), and (c) d_3 to the diagonal directions called octahedral (OCTA). Other stencils can be configured by combination of these stencils. The accuracy is controlled by adjusting these parameters as shown in Table 1. The cut-off frequency is obtained from the maximum value of the normalized frequency that is calculated by the numerical dispersion curve for the representative propagation directions. For example, the cut-off frequency for the diagonal direction is given as

$$\bar{f}_c = \frac{\sin^{-1}(\chi\sqrt{3-12a+16b})}{\pi} \quad (3)$$

Figure 2 shows the cut-off frequency against the CFL number for the diagonal direction. The

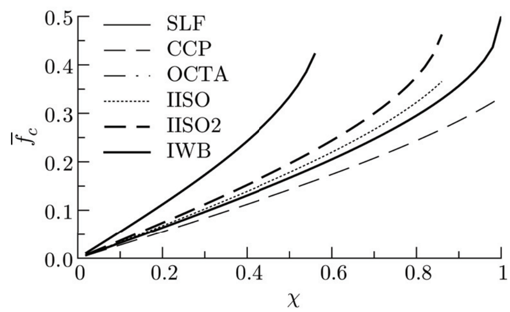

Fig. 2 Cut-off frequency against CFL number.

cut-off frequency becomes maximum when the CFL number is maximum. Therefore the computer resources depend on the cut-off frequency and the CFL number.

3. Numerical experiments

3.1 In the case of single GPU

The CE-FDTD method is implemented on the single GPU (NVIDIA Tesla M2075). The calculation performance is evaluated for a numerical model of a cubic room with $1m^3$. The discretization is made to achieve the cut-off frequency of 20kHz for each scheme. An impulse point source is located at the center of the room. All boundary conditions are assumed to be rigid. The memory usage and the calculation time for the impulse response with 1 second long are measured using each scheme as shown in Fig. 3. In the figure, the measured values are normalized by the results by SLF. It is found that the memory usage of the IWB scheme is smallest and is about 30 % compared with the SLF. It is also found that the calculation time with the shared memory becomes shorter than that without the shared memory. This is because the memory transfer is optimized using shared memory. It is also confirmed that the calculation time of the IWB scheme is shortest and is about 14% compared with SLF.

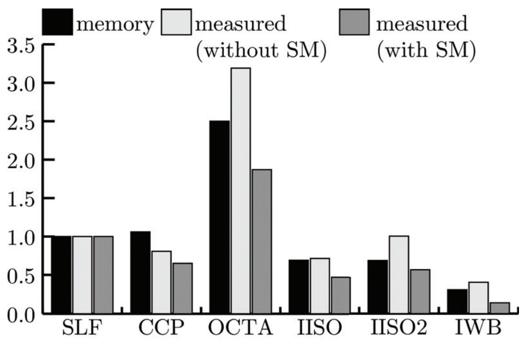

Fig. 3 Comparison of computational performance with consideration of the accuracy in the case of single GPU.

3.1.2 In the case of GPU cluster

Next we discuss the case of the GPU cluster system. The developed GPU cluster system consists of 8 PC nodes. Four GPUs are mounted in each node, so the total number of GPUs is 32 in the system (M2075×32, 192GB total memory). For the parallel calculation, the domain is divided into the virtual sub domains along *x*-direction. The calculation performance is again evaluated for a

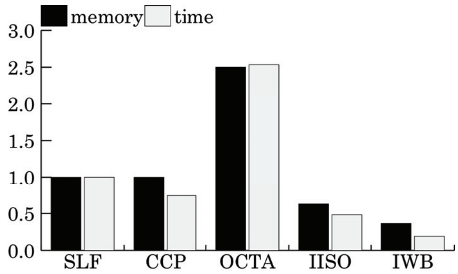

Fig. 4 Comparison of computational performance in the case of GPU cluster system.

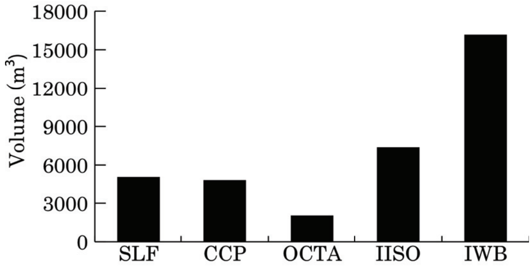

Fig. 5 Maximum volume capacity of the numerical room model when the full memory of the cluster (192MB) is used.

numerical model of a cubic room with $10m^3$. It is confirmed in the case of the GPU cluster system that the memory usage of the IWB scheme is almost 36 % and the calculation time is almost 19% compared with SLF. The performance with the GPU cluster system rather degrades than the single GPU because it takes time to transfer virtual boundary data between GPUs by the PCI Express bus or the InfiniBand link.

Fig. 5 shows the maximum volume capacity of the numerical room model when the full memory of the cluster (192MB) is used under the condition of the cut-off frequency of 20kHz. The largest capacity is achieved for the IWB scheme, which is three times lager than the SLF scheme. The impulse response of the large room is finally calculated by the IWB scheme with the developed cluster system. The room capacity of 23.93 m × 30.46 m×19.58 m (≒ $14272m^3$) is assumed. In this case, the calculation time was 26.7 hours. It is confirmed that the IWB scheme is the best algorithm when the required cut-off frequency is given.

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