1P1-9

Proceedings of Symposium on Ultrasonic Electronics, Vol. 34 (2013) pp. 31-32
20-22 November, 2013

Impulse Response for Electromechanical Coupling System on A Distributed-Parameter Basis

分布定数回路的な手法による電気機械結合系のインパルス応答の計算方法

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1. Introduction

The transient response analysis (the calculation of impulse response function) is important as well as the frequency characteristic analysis. Although the frequency characteristics have been widely utilized in the classical theory, the importance of the time series analysis has been increased in recent years for 'modern' methodologies.

There are two types of representation methods for a system----one is lumped-parameter-based and the other is distributed-parameter-based. In order to analyze an elastic vibration system or an electromechanical coupling system, the author has developed the framework of the analysis including only distributed parameters without any lumped-parameter components. [1-4]

In this paper, the principle of analyzing the impulse response for an electromechanical coupling system using this framework----termed 'complex series dynamics'----is discussed, and the advantage of this method over the conventional lumped-parameter-based framework is also discussed.

2. Neumann series in complex series dynamics

In the framework of the complex series dynamics, the characteristic function is given by the probabilistic superposition of the infinite geometric series of matrices, 'Neumann series', in which the initial term and the common ratio reflect the boundary conditions of the system. The size of the matrix is N by N in the case of a one-dimensional elastic vibration system and $2N$ by $2N$ in the case of a system with electromechanical coupling, where N is the number of spatial layer in the system.

The characteristic function is given by

$$\boldsymbol{\eta}(\omega) = \sum_j K_{\mathrm{out}(j)} \left(A_{0(j)} + R_{(j)} A_{0(j)} + {R_{(j)}}^2 A_{0(j)} + \ldots \right) \times K_{\mathrm{in}(j)} \boldsymbol{\eta}_0, \qquad (1)$$

where $\boldsymbol{\eta}_0$ and $\boldsymbol{\eta}$ are an initial seed value and a final resultant value, respectively, of energy mode---elastic mode with N elements and dielectric mode with N elements, $2N$ elements in total, and $K_{\mathrm{in}(j)}$, $K_{\mathrm{out}(j)}$, $A_{0(j)}$, $R_{(j)}$ (j is the label for distinguishing probabilistic paths) are matrices that reflect the boundary conditions of the system.

Equation (1) provides frequency characteristics of the system as a function of ω, where ω is an angular frequency of the energy mode as a wave. (Both of $A_{0(j)}$ and $R_{(j)}$ are functions of ω.)

In eq. (1), ω is a definite value, while the time t is indefinite, since infinite numbers of events that occur at different times are superposed into one. From the viewpoint of the uncertainty principle between frequency and time, indefinite time causes definite frequency.

In ref. 2, the impulse response of elastic mode without electromechanical coupling with dielectric mode was obtained as the following time series:

$$\begin{aligned}\boldsymbol{\eta}(t) = (&\sum_j K_{\mathrm{out}(j)} A_{0(j)}|_{\omega=0} K_{\mathrm{in}(j)} \boldsymbol{\eta}_0, \\ &\sum_j K_{\mathrm{out}(j)} R_{(j)}|_{\omega=0} A_{0(j)}|_{\omega=0} K_{i(\mathrm{in})} \boldsymbol{\eta}_0, \\ &\sum_j K_{\mathrm{out}(j)} {R_{(j)}}^2|_{\omega=0} A_{0(j)}|_{\omega=0} K_{\mathrm{in}(j)} \boldsymbol{\eta}_0, \\ &\ldots),\end{aligned} \qquad (2)$$

where ω was set to zero in $A_{0(j)}$ and $R_{(j)}$.

All of the resonance modes that can be driven are active (stiffness control) in the low frequency limit, and therefore, the situation of ω = 0 in the elastic mode is regarded as the situation of indefinite frequency, which causes definite time, according to the uncertainty principle.

Equation (2) is 'discrete' in time domain, which means that the characteristic of the system is 'periodic' in frequency domain, according to the fundamental theorem of Fourier transform. In other words, the method described with eq. (2) cannot be adopted in the case of 'aperiodic' frequency characteristics. For example, when the electromechanical coupling occurs, the frequency characteristic function of the system is not periodic any longer in the frequency domain, since the effect of coupling phenomenon is decreased as the frequency becomes higher. The same can be said of the situation in which the dissipation of the system has frequency dependence. The aperiodic frequency characteristic function should cause a continuous impulse response.

3. Resolvent and its application

Equation (1) is equivalent to

$$\boldsymbol{\eta}(\omega) = \sum_j K_{\mathrm{out}(j)}(I - R_{(j)}(\omega))^{-1} A_{0(j)}(\omega) K_{\mathrm{in}(j)} \boldsymbol{\eta}_0, \quad (3)$$

where I is a unit matrix whose diagonal elements are ones and the other elements are zeros, and eq. (3) includes a 'resolvent' in linear algebra in mathematics. Usually, a resolvent $r(\lambda)$ is defined as a function of λ:

$$r(\lambda) = (\lambda I - R)^{-1} \quad (4)$$

for a matrix R. Apparently, the resolvent is equivalent to Neumann series. When the determinant of $r(\lambda)$ is infinite:

$$\det(r(\lambda)) \to \infty, \quad (5)$$

λ gives the eigenvalue of R, from which the eigenvector is also determined.

In this study, this concept is used to obtain the impulse response of a system with electromechanical coupling.

The value of ω, at which

$$\det((I - R_{(j)}(\omega))^{-1}) \to \infty, \quad (6)$$

corresponds to the eigenvalue (not real number, but complex number) of the system, ω_n. The eigenvector of the system with spatial N elements, $\boldsymbol{\eta}_n$, is also obtained for each of ω_n. n is the label for an eigenstate. Then, the impulse response of the system is simply given by

$$\boldsymbol{\eta}(t) = \sum_n a_n \boldsymbol{\eta}_n \exp(\omega_n t), \quad (7)$$

where $\boldsymbol{\eta}(t)$ is the impulse response of elastic mode with spatial N elements under the electromechanical interaction with dielectric mode, a_n is a scalar indicating the degree of contribution of initial distribution of $\boldsymbol{\eta}(t)|_{t=0}$ to the eigenvector $\boldsymbol{\eta}_n$.

Practically, the complicated calculation of the determinant is not necessary, and from the peak value of the frequency characteristic function $\boldsymbol{\eta}(\omega)$, ω_n and $\boldsymbol{\eta}_n$ can be determined, and a_n is also determined from the spatial inner product between $\boldsymbol{\eta}_n$ and $\boldsymbol{\eta}(t)|_{t=0}$.

4. Comparison with lumped-parameter methods

The frequency characteristic function of the lumped-parameter circuit for an electromechanical coupling system has a pair of resonance and antiresonance in each resonance mode. At the antiresonance frequency, the admittance is reduced to the minimun, but the vibration level is not reduced as long as the electric source is connected to the system. That is, the situation at the antiresonance frequency is actually not 'antiresonance' from the viewpoint of energy, ----This can be shown experimentally, for example, by measuring the vibration level using a laser-optic method. [5] ----and therefore, the admittance does not represent the situation of vibration level and energy correctly. Therefore, the inverse Fourier transform of the admittance cannot give the impulse response of the vibration correctly, which is a harmful side effect of lumped-parameter dielectric capacitance C_0, although the existence of C_0 is inevitable to express the shift of acoustic speed due to the electromechanical coupling.

When the Q-value of the system is not so large, C_0 also causes the difficulty in estimating the resonance peak value and its frequency (an eigenvalue of the system). Although this difficulty can be reduced numerically and statistically, [6,7] the errors cannot be removed completely.

The present method can evaluate 'local' transient response, while the lumped-parameter-based ones cannot. (In order to improve the spatial resolution of eigenvector, the value of N should be increased, according to the Nyquist theorem applied to the spatial domain.)

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