

# Guided Waves Propagating in a Water-Filled Stainless Steel Pipe

水を満たした SUS パイプを伝搬するガイド波

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## 1. Introduction

Cylindrical pipes are widely used in industries such as nuclear power plants and micro total analysis systems ( $\mu$ TAS). Nondestructive evaluation (NDE) of such pipes is therefore crucial. NDE as well as ultrasonic flowmeters can be used to characterize pipes filled with fluid. Guide wave of a hollow pipe was investigated theoretically by Gazis<sup>1</sup>, and we previously expanded on the theory proposed by Gazis for a fluid-filled pipe<sup>2,3</sup>. Our theoretical results can apply to all mode including non-axisymmetrical modes. The investigations of guided waves in fluid-filled pipes for axisymmetric mode exist, but not so much. Then, we did theoretical and experimental study of guided wave in a fluid-filled pipe in this article.

## 2. Theoretical analysis and results

Fig. 1 shows the theoretical model of a cylindrical pipe and its coordinate system (cylindrical coordinates). The author's theoretical basis is an expansion of that of hollow pipe by Gazis<sup>1</sup>. When those attenuations are negligible small, the displacement  $\mathbf{u}^{\text{solid}}$  of the pipe ( $a \leq r \leq b$ ) and the displacement  $\mathbf{u}^{\text{fluid}}$  of a fluid ( $0 \leq r \leq a$ ) are represented as follows.

$$\begin{aligned} u_r^{\text{solid}} &= [f'_s + \frac{n}{r} g_3 - kg_1] \cos n\theta \cos(\omega t - kz) \\ u_\theta^{\text{solid}} &= [-\frac{n}{r} f_s - kg_1 - g'_3] \sin n\theta \cos(\omega t - kz) \\ u_z^{\text{solid}} &= [kf'_s - g'_1 - \frac{n+1}{r} g_1] \cos n\theta \sin(\omega t - kz) \\ u_r^{\text{fluid}} &= f'_f \cos n\theta \cos(\omega t - kz) \\ u_\theta^{\text{fluid}} &= -\frac{n}{r} f_f \sin n\theta \cos(\omega t - kz) \\ u_z^{\text{fluid}} &= kf_f \cos n\theta \sin(\omega t - kz) \end{aligned} \quad (1)$$

Here, the author consider acoustic waves that propagate along the z-direction and whose angular frequency is  $\omega$  and wave number is  $k$ .  $n$  is the circumferential mode parameter.  $f_s$ ,  $g_r$ ,  $g_\theta$ ,  $g_3$ , and  $f_f$  are represented by a linear combination of Bessel functions.<sup>2</sup> Those coefficients are  $A$ ,  $A_1$ ,  $A_3$ ,  $B$ ,  $B_1$ ,  $B_3$ , and  $A_f$ , and they are decided to

satisfy boundary conditions below.

$$\begin{aligned} u_r^{\text{solid}} = u_r^{\text{fluid}}, \quad \sigma_{rr}^{\text{solid}} = \sigma_{rr}^{\text{fluid}}, \quad \sigma_{r\theta}^{\text{solid}} = \sigma_{r\theta}^{\text{fluid}} = \sigma_{rz}^{\text{solid}} = \sigma_{rz}^{\text{fluid}} = 0 \quad \text{at} \\ r = a \\ \sigma_{rr}^{\text{solid}} = \sigma_{r\theta}^{\text{solid}} = \sigma_{rz}^{\text{solid}} = 0 \quad \text{at} \quad r = b \end{aligned} \quad (2)$$

$\sigma^{\text{solid}}$  and  $\sigma^{\text{fluid}}$  represent the stress tensors of a solid and fluid, respectively. Eq. (2) is represented by a 7 dimensional simultaneous linear equations as follow.

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & c_{17} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} & c_{27} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} & c_{37} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} & c_{47} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} & c_{57} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} & c_{67} \\ c_{71} & c_{72} & c_{73} & c_{74} & c_{75} & c_{76} & c_{77} \end{pmatrix} \begin{pmatrix} A \\ A_1 \\ A_3 \\ B \\ B_1 \\ B_3 \\ A_f \end{pmatrix} = 0 \quad (3)$$

Here,  $c_{ij}$ s are calculated by frequency ( $f$ ), phase velocity ( $V$ ) and so on.<sup>2</sup> In order to obtain  $V$ , we should seek a phase velocity which satisfies a determinant formula  $|c_{ij}| = 0$ . In order to obtain displacements, we should solve eq. (3) and substitute the solution into eq. (1).

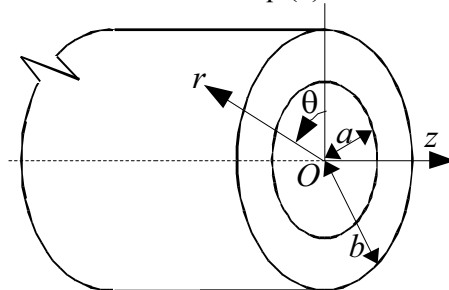


Fig.1 Analytical Model

The parameters used in the calculation are as follows:

$$\begin{aligned} v_i &= 5790 \text{ m/s}, \quad v_r = 3100 \text{ m/s}, \quad \rho_s = 7910 \text{ kg/m}^3 \\ v_f &= 3100 \text{ m/s}, \quad \rho_f = 1000 \text{ kg/m}^3 \end{aligned}$$

Red and blue curves of Fig.2 are theoretical results of L-mode<sup>2</sup> (axisymmetric mode) and  $n=1$  (non-axisymmetric mode), respectively.

## 3. Experimental results and discussions

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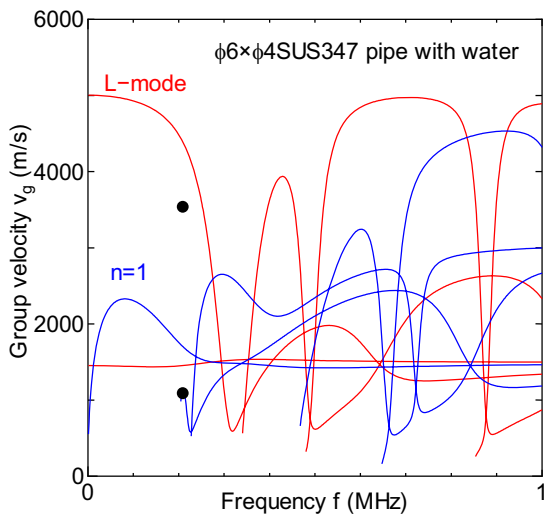
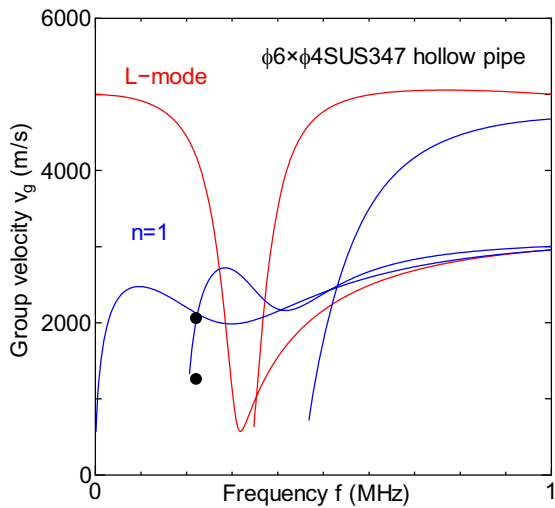


Fig.2 Group velocities of Guided waves propagating in a hollow SUS pipe or a water-filled SUS pipe

By Fig. 2, the specific impedance of SUS is much larger than that of water, but the effect of water appears. In order to verify this theoretical result, we did experimental study. Fig. 3 shows experimental setup. We used  $\phi 6 \times \phi 4$  SUS304 pipe (Fig. 4). A upper and lower figures of Fig. 5 show guided waves propagating in a hollow SUS pipe or a water-filled SUS pipe, respectively. Black curves of Fig.5 shows RF signals and blue curve shows a wavelet transform of a RF signal at 222 kHz, and red curve shows a wavelet transform of a RF signal at 210 kHz. The RF signals differs clearly between hollow pipe and water-filled pipe. By using the maximum peak of wavelet transform, we obtained propagating velocities of guided waves (Fig.2 black circles). Those results are agreed with theoretical results.

**Acknowledgment**

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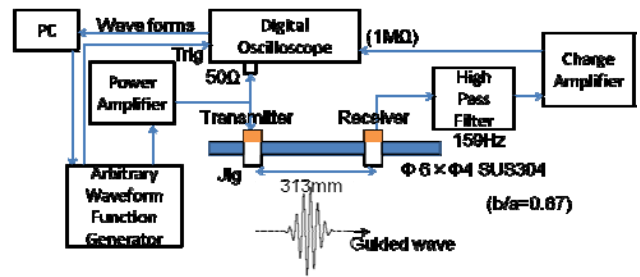


Fig.3 Experimental setup



Fig.4 Picture of experiment

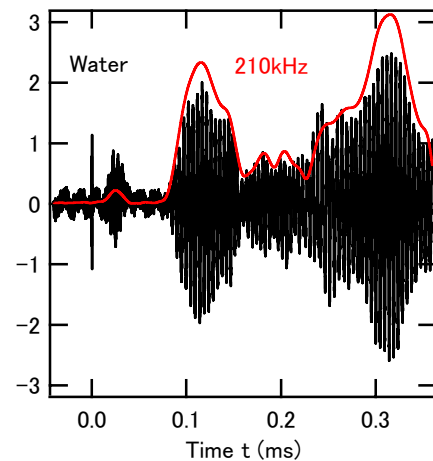
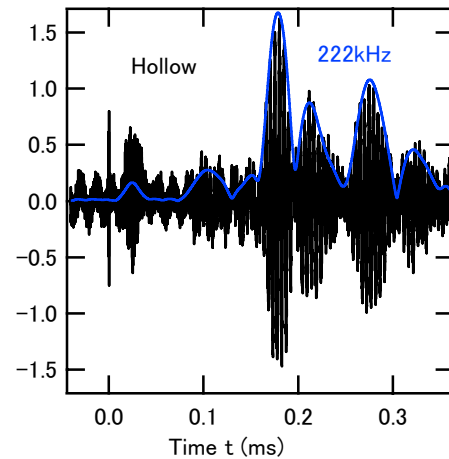


Fig.5 Guided waves propagating in a hollow SUS pipe or a water-filled SUS pipe

**References**

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2. H. Sato, M. Lebedev and J. Akedo: Jpn. J. Appl. Phys. **45** (2006) 4573.
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