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Analysis of Change in Motional Capacitance of Quartz-Crystal Tuning-Fork Tactile Sensor Induced by Viscoelastic Materials in Contact with Its Base

音叉型水晶触覚センサの基底部に接触した粘弾性物体で誘起される動的容量変化の解析

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Abstract- We have found that the change in the reciprocal of motional capacitance, before and after a quartz-crystal tuning-fork tactile sensor's base getting into contact with neoprene rubbers, has been intrinsically induced by both the dynamic Young's modulus and viscosity of neoprene rubbers at the resonance frequency 32.5 kHz of the quartz tactile sensor.

1. Introduction

As far as we know, our vibrating tactile sensor using the change in motional capacitance [1,2] is the first one. We have already showed that the change in the reciprocal of its motional capacitance $\Delta(1/C_a)$ before and after the sensor's base getting into contact with materials yields with the Young's modulus of materials.[2]

In this study, we investigate theoretically and experimentally that the change in the motional capacitance of the quartz-crystal tuning-fork tactile sensor varies with both the dynamic Young's modulus and viscosity of amorphous polymers such as neoprene rubbers.

2. Analysis

Figures 1(a) and 1(b) show the configuration of the right half of the quartz-crystal tuning-fork and a torsion spring which is introduced at the joint of its base and arm, respectively. In Fig. 1(a), we approximate the base as flexural beam A and the arm as flexural beam B. the equation of motion depicted by Voigt viscoelastic body for the base in contact with viscoelastic materials and the equation of motion for the arm are given by

$$E_1 I_1 \frac{\partial^4 u_1}{\partial x_1^4} + P_t \frac{\partial^2 u_1}{\partial x_1^2} + A_1 \rho \frac{\partial^2 u_1}{\partial t^2} + \mu \frac{\partial u_1}{\partial t} + k u_1 = 0, \quad (1)$$

$$K_c \frac{\partial^4 u_2}{\partial x_2^4} + A_2 \rho \frac{\partial^2 u_2}{\partial t^2} = 0, \quad (2)$$

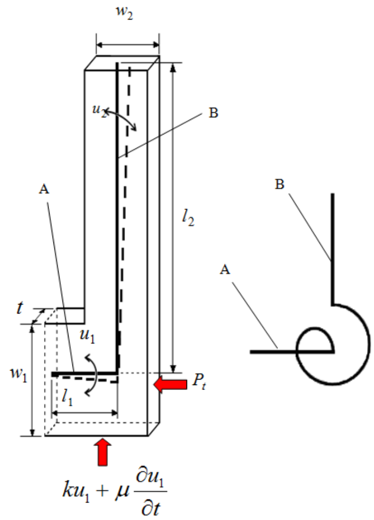

Fig.1. (a) Configuration of the L-shaped right half of the quartz-crystal tuning-fork tactile sensor and (b) a torsion spring at the joint of beams A and B.

where E_1 is the Young's modulus of the base, I_1 is the moment of inertia for the base, A_1 is the cross-sectional area of the base, A_2 is the cross-sectional area of the arm, ρ is the mass density of quartz crystal, P_t is the lateral clamping force from the acrylic resin case, k is the Winkler's foundation coefficient, μ is the viscosity coefficient of viscoelastic materials

The flexural displacement u_1 for the base and u_2 for the arm are assumed to have the same time term $z_n(t)$. Accordingly, u_1 and u_2 are given using the method of separation of variables by

$$u_1 = \sum_n z_n(t) \cdot \phi_{n1}(x_1), \quad u_2 = \sum_n z_n(t) \cdot \phi_{n2}(x_2), \quad (3)$$

where n is the vibrational mode number, and ϕ_{n1} and ϕ_{n2} are characteristic function of the beams A and B, respectively.

Using conservation law of energy P_1(electrostatic energy in motional capacitance)-P_4(loss energy of the materials in contact with its base)=P_2(strain energy of the vibrating quartz arm)+P_3(stored energy of the materials in contact with its base), eventually we can obtain the following motional capacitance formula including the viscosity of viscoelastic materials as

$$C_a = \frac{2Q^2}{K_c \int_0^{l_2} \left(\frac{\partial^2 u_2}{\partial x_2^2} \right)^2 dx_2 + k \int_0^{l_1} u_1^2 dx_1 + 4P_4}. \qquad (4)$$

The loss energy P_4 for the viscosity term in eq. (1) is given by

$$P_4 = E'' \int_0^{l_1} \left(\int_0^{\frac{2\pi}{\omega_n}} \left(\frac{\partial}{\partial t} (z_n \cdot \phi_{n1}) \right)^2 dt \Big/ \left(\frac{2\pi}{\omega_n} \right) \right) dx_1, \qquad (5)$$

where E'' is the loss modulus.

3. Experiments and discussions

In **Fig. 2,** the calculated values 1, subtracting the calculated reciprocal of eq. (4) without P_4 term by use of the values of dynamic Young's moduli of neoprene rubbers at 32.5 kHz obtained using DMA measurement are indicated by closed marks with respect to dynamic viscosity of neoprene rubbers at 32.5 kHz. The calculated values 2, subtracting the calculated reciprocal of eq. (4) without P_4 term by use of the same values of neoprene rubbers described above from the calculated reciprocal of eq. (4) in which P_4 given by eq. (5) is calculated at n=1 by use of the values of dynamic viscosity of neoprene rubbers at 32.5 kHz, are also indicated by open marks at the same time in Fig. 2. In Fig.2, the open and closed marks seems to change with viscosity. Accordingly, figure 2 shows clearly that A part of $\Delta(1/C_a)$ due to viscosity changes with only the dynamic viscosity of neoprene rubbers.

4. Conclusion

We have derived the analytical formula for the motional capacitance of the quartz-crystal tuning-fork tactile sensor using L-shaped bar's and viscoelastic foundation models, and conservation low of energy between the electrostatic energy in motional capacitance and the sum of the strain energy of the vibrating quartz arm, the stored and

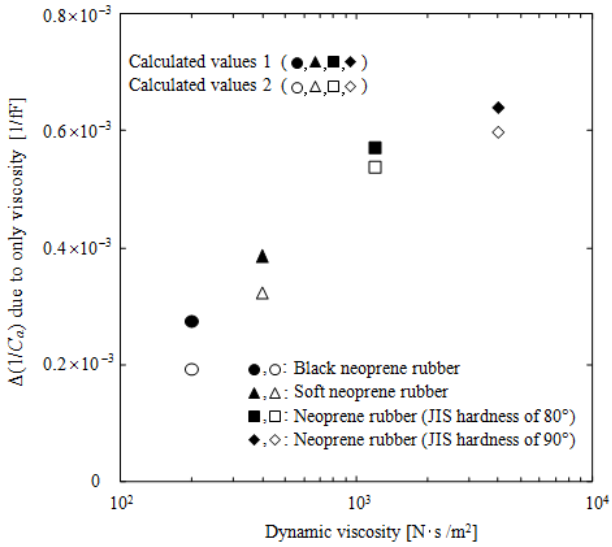

Fig.2. A part of $\Delta(1/C_a)$ due to only viscosity vs dynamic viscosity of neoprene rubbers. A part of $\Delta(1/C_a)$ due to only viscosity means two values: one, indicated by open marks, is the calculated value subtracting the reciprocal of eq. (4) without P_4 term from the reciprocal of eq. (4), another, indicated by closed marks, is the calculated value subtracting the reciprocal of eq. (4) without P_4 term from the measured value of $\Delta(1/C_a)$.

loss energies of the materials in contact with its base. Our theoretical and experimental examinations that the change in the reciprocal motional capacitance before and after the sensor's base getting into contact with neoprene rubbers has been quantitatively yielded by the effect of both their dynamic Young's modulus and viscosity of them at 32.5 kHz have been established.

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References

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