# **Analysis of passive Surface Acoustic Wave sensors using Coupling of Modes theory**

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### **1. Introduction**

Surface acoustic wave (SAW) devices using a piezoelectric clystal have been applied to not only filters but also sensors. Recently, the measurement method which is combination of a SAW device and classical sensor for detecting physical quantity, namely a passive SAW sensor, has been studied. The measurement system has advantages of no power supply and electric circuit for sensors. The system greatly simplifies usual measurements. In this paper, we present the modeling for passive SAW sensors using coupling of modes (COM) theory. Then we propose the applictaion based on measurement principle of passive SAW sensors.

## **2. Coupling of modes for uniform passive SAW sensors**

We adopted the COM formulation[1, 2]. The COM equations governing SAW modes amplitudes  $A_+(x)$  and  $A_-(x)$  propagating in the  $\overline{+x}$  and  $-x$  directions (**Fig. 1**) can be written as

$$
\frac{\partial A_+(x)}{\partial x} = -j\theta_u A_+(x) - j\kappa_{12} A_-(x) + j\zeta V,
$$
  

$$
\frac{\partial A_-(x)}{\partial x} = jk_{12}^* A_+(x) - j\theta_u A_-(x) - j\zeta V,
$$
  

$$
\frac{\partial I(x)}{\partial x} = -4j\zeta A_+(x) - 4j\zeta A_-(x) + j\omega C_s V,
$$
  

$$
\theta_u = 2\pi(\frac{f l}{V_{SAW}} - 1) + \kappa_{11}.
$$

Where  $\kappa_{11}$ ,  $\kappa_{12}$  and  $\zeta$  are coefficient of self-coupling, coupling of modes and transduction, respectively.  $C_s$  is static capacitance per unit length and  $V_{SAW}$  is phase velocity of SAW. Those are called COM parameters and very important. Those are depended on the direction of propagation and electrode construction and calculated using the finite element method[1]. In this paper, first, we used literature values of the parameters[1]. To facilitate the cascading of uniform transducer elements, solution of the COM equations can be presented in the P matrix form. For the passive SAW sensor, we pay attention to the reflector which is connected to the classical sensor. For the reflector, we define P matrix as follows,

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$$
\begin{bmatrix} A_{-}(0) \\ A_{+}(L) \\ -Y_{cs}V \end{bmatrix} = \begin{bmatrix} P_{11}^r & P_{12}^r & P_{13}^r \\ P_{21}^r & P_{22}^r & P_{23}^r \\ P_{31}^r & P_{32}^r & Y_{cs} + P_{33}^r \end{bmatrix} \begin{bmatrix} A_{+}(0) \\ A_{-}(L) \\ V^r \end{bmatrix} \quad (2)
$$

In eq. (2),  $Y_{cs}$  is admittance of the classical sensor and  $A_{(L)} = 0$  is boundary condition. For a wave propagating from the IDT, reflection coefficient  $\Gamma$ , is defined as follows.

$$
\Gamma_r = \frac{A_{-}(0)}{A_{+}(0)} = P_{11}^r + \frac{P_{13}^r P_{31}^r}{-2Y_{cs} - P_{33}^r}.
$$
 (3)

Using  $\Gamma_r$ , the wave propagating from the reflector to the IDT is expressed as,

$$
A_{-}(L) = \Gamma_{r} e^{-2j\beta L} A_{+}(L), \tag{4}
$$

 $A_{-}(L) = \Gamma_{r} e^{-2j\beta L} A_{+}(L)$ ,<br>where  $\beta$  is wave number and L is propagation distance. P matrix for IDT is presented as  $_{\beta}$ 

$$
\begin{bmatrix} A_{-}(0) \\ A_{+}(L) \\ I^{I} \end{bmatrix} = \begin{bmatrix} P_{11}^{I} & P_{12}^{I} & P_{13}^{I} \\ P_{21}^{I} & P_{22}^{I} & P_{23}^{I} \\ P_{31}^{I} & P_{32}^{I} & P_{33}^{I} \end{bmatrix} \begin{bmatrix} A_{+}(0) \\ A_{-}(L) \\ V^{I} \end{bmatrix}
$$
 (5)

Admittance Y of the passive SAW sensor is conducted using eq. (4) and  $A_+(0) = 0$  which is boundary condition.

$$
Y = \frac{I^I}{V^I} = \frac{P_{23}^I P_{32}^I}{\left[\Gamma_r e^{-2j\beta L}\right]^{1} - P_{22}^I} + P_{33}^I.
$$
 (6)

Using eq. (6), S11 is obtained.

#### **3. Comparison between simulated and measured results**

We used a  $128°*YX* – *LiNbO*<sub>3</sub> piezoelectric$ substrate. Operating frequency of the passive SAW sensor was 50.5MHz. The passive SAW sensor used is shown in **Fig. 2**. The propagated distance was 8mm. The apeture of transducer was 2.96mm. Firstly, the simulated and measured results of conductance and susceptance were compared. Difference of the waveform shapes were found. In

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**Fig. 3,** measured time response (solid line) and simulated one (dashed line) are compared. Both results do not agree. In Fig. 4, measured  $(\triangle)$  and simulated  $\circ$ ) results of first echo amplitude as a function of impedance are shown. The results do not agree.







Fig. 4 The relation between impedance and amplitude.

#### **4. Determination of COM parameters and result**

We have to determine accurate COM parameters to obtain precise calculation results[2]. In this paper, a method presented by Brent[3] which is in combination of the parabolic interpolation and golden section method was adopted to obtain correct parameters. This method enables us to lead optimum solution in the arbitrary range. The objective function  $F$  is defined as follows,

$$
F = \sum_{i=1}^{N} \left\{ (C_s(i) - C_m(i))^2 + (S_s(i) - S_m(i))^2 \right\}, (7)
$$

where  $C_s$  and  $S_s$  are simulated result of conductance and susceptance and  $C_m$  and  $S_m$  are measured ones, *i* is the discrete frequency and *Cs <sup>s</sup> S*

*N* is the number of date points. We used optimized COM parameters which makes simulated results near the measured ones for arbitrary range. Then, COM parameters were optimized as  $\kappa_{11}$  is 0.192,  $\kappa_{12}$  is 0.0399,  $\zeta$  is 0.00799,  $C_s$  is 400e-12[pF/m],  $V_{SAW}$  is 3999.7[m/s]. Using optimum COM parameters, simulated results agree with experimental results (see **Figs. 3** (dotted line) and  $4 \, (\Box)$ ).

#### **5. Application**

 To identify SAW devices, different structures of reflectors have been used. In this paper, however, we propose new method, using impedance of  $1\Omega$ or  $10k \Omega$ . From Fig. 4, difference of echo amplitudes for 1  $\Omega$  and 10k  $\Omega$  are large. Identification of SAW sensors will be achieved using the difference. Simulated device structure has two reflectors and one IDT. Distances between reflectors and IDT were L/2 and L, respectively. Simulated results are shown in **Fig. 5**. The programmable 2 bit ID SAW device can be realized.



Fig. 5 The results of the new ID SAW device.

#### **6. Conclusion**

In this paper, we discussed modeling for passive SAW sensors. By using optimum COM parameters, precise results were obtained. Then, the programmable ID SAW device was proposed. We have only to make one structure for the ID device. This report may enable us to simulate all things about using operating principles of passive SAW sensors.

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#### **References**

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