

## Evaluation of Local Velocity Mapping for Nonlinearity of Shear Elasticity

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### 1. Introduction

Tissue mechanical characterization using shear wave velocity has been the one of the most important expectations in the current medical diagnosis system. We have proposed a novel method for estimating the velocity of the shear wave propagation inside the tissue by using continuous shear wave excitation (CSWE)[1,2]. We have also proposed an algorithm for reconstruction of harmonic phase modulation components in order to characterize the different mechanical properties of the tissue [3].

Lots of theoretical frameworks or models have been introduced in order to model the nonlinear or hysteretic characteristics of the shear wave propagation [4,5]. Also many researchers had studied about the presence of Hookean behavior in the human body components [6]. But no such sophisticated model has been introduced till, which can explain the exact form of the hysteresis by using equations. Two different types of damping; viscous damping: a linear damping, and hysteretic damping: a nonlinear damping, are present in a system [7]. Viscous damping is proportional to the frequency of vibration and has no effect for zero frequency. On the other hand, hysteretic damping is less dependent to the frequency of vibration and, for this reason, nonlinear characteristic of shear elasticity for very low frequency is closely relates with the hysteretic damping. Many imaging method has been proposed for viscous characteristics of the shear elasticity [8], but only a few methods have been proposed for hysteretic characterization.

In this paper, we propose a model that characterizes the nonlinearity i.e. the hysteretic behavior of the shear elasticity. Our model assumes small harmonic phase modulation of displacement in shear wave propagation in which the hysteresis is independent of vibration frequency. In this static model, hysteresis for dc components is measured and is compared with texture pattern of the local velocity map.

### 2. Harmonic Phase Modulation of Displacement as a Model of Nonlinearity of Shear Elasticity

We have already proposed a novel imaging method which uses the local velocity of shear wave propagation to reconstruct the phase modulation component [3]. Let us consider a model of hysteresis of shear elasticity based on the displacement of shear wave due to small harmonic phase modulation component which represented by the following equation.

$$s(t) = A_0 \sin[\omega_0 t + m \sin(n\omega_0 t + \varphi)] \quad (1)$$

Where,  $A_0$  is the displacement amplitude of displacement,  $\omega_0$  is the fundamental angular frequency,  $m$  is the modulation index and  $n$  is the order of harmonic component.

We define two parameters,  $P_p$  as a parameter of nonlinearity of displacement wave and  $P_0$  as a parameter of hysteresis of displacement wave. By using these parameters, we illustrate the hysteretic behavior of the displacement shear wave due to small phase modulated harmonic components. Fig.1 shows the hysteresis loop due to presence of phase modulated harmonic component represented by eq. (1).

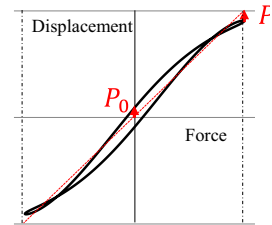


Fig.1. Hysteresis due to presence of phase modulated harmonic components.

At static condition, at time  $t = 0$ , and for very small phase modulation index  $m \cong 0$ ,

$$s(0) = A_0 \sin(m \sin \varphi) \cong A_0 m \sin \varphi \quad (2)$$

Parameter of hysteresis of displacement wave is:

$$P_0 = m \sin \varphi \quad (3)$$

The eq. (3) shows that hysteresis of the displacement wave increases with the increase in phase modulation index.

Applying the boundary condition for the parameter of nonlinearity as  $\omega_0 t = \frac{\pi}{2}$  i.e.  $t = 1/(4f_0)$  for very small phase modulation index of  $m \cong 0$ ,

$$s\left(\frac{1}{4f_0}\right) \cong A_0 \left[1 - \frac{1}{2} m^2 \sin\left(\frac{n\pi}{2} + \varphi\right)^2\right] \quad (4)$$

Parameter of nonlinearity of displacement wave is:

$$P_p = \frac{1}{2} m^2 \sin\left(\frac{n\pi}{2} + \varphi\right)^2 \quad (5)$$

The eq. (5) shows that the parameter  $P_p$  is proportionally dependent on the three parameters: phase modulation index  $m$ , harmonic order  $n$  and the phase  $\varphi$ . The simulation result shown in Fig.2 illustrates the dependency of hysteretic characteristics of the shear wave elasticity with respect to  $m$  and  $\varphi$ . In case of change in harmonic order  $n$ , the symmetry of the hysteresis loop changes according to even and odd harmonic order.

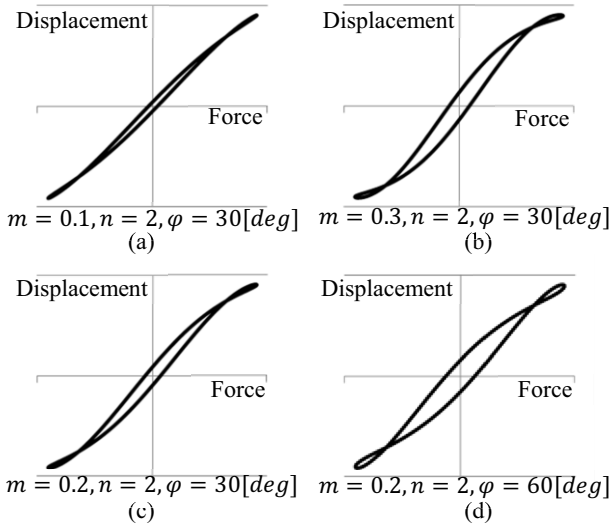


Fig.2. Dependency of hysteretic characteristics. (a): Nonlinearity due to phase modulation index. (b): Change in nonlinearity due to change in phase modulation index. (c) and (d): Change in nonlinearity due to change in phase.

### 3. Experimental Results

The texture pattern seen in the local velocity mapping due to phase modulation component [3] also depends on mean of local velocity within the local velocity map. The mean local velocity what we called a normalized shear wave velocity which is defined by the ratio of actual velocity to the average velocity, is introduced in order to evaluate nonlinearity of the medium. Experiments were carried out on carrageenan gel. Under the quasi-static condition, carrageenan gel of different wt% (0.5wt%, 0.9wt%, 1.3wt%) are examined to evaluate the stress-strain curve using the rheometer (Shimazu, Kyoto, Japan) under the constant loading pressure of around 3N. Also the nonlinearity of the shear wave is measured by the skewness of the normalized local velocity map, which is an index of asymmetry of image brightness. Fig.3. shows the skewness vs. hysteresis plot. The correlation analyzed between the hysteresis and the skewness is found to be 0.97. The obtained correlation value shows the high affinity

of the proposed method to evaluate the nonlinearity of shear wave elasticity.

### 4. Conclusions

We have proposed a novel model of hysteresis which uses the harmonic phase modulated components of shear elasticity to evaluate the nonlinearity of shear elasticity. By using this model we discuss the relationship of the modulation index, harmonic order and the phase with the hysteretic or the nonlinearity of the medium. By changing these parameters, the nonlinearity or hysteresis can be simulated. Though the simulation result is a preliminary one, once the model is validated, it may become a powerful tool to relate the viscoelasticity with the hysteretic or the nonlinearity of shear elasticity.

Using the local velocity mapping method, which we have already proposed, we carried out the experiments on carrageenan gel. From the result, the nonlinearity of carrageenan gel and skewness of the texture pattern is found to have good correlation between them. This shows that the local velocity mapping is a novel method in order to reconstruct the nonlinearity or the hysteresis of shear elasticity.

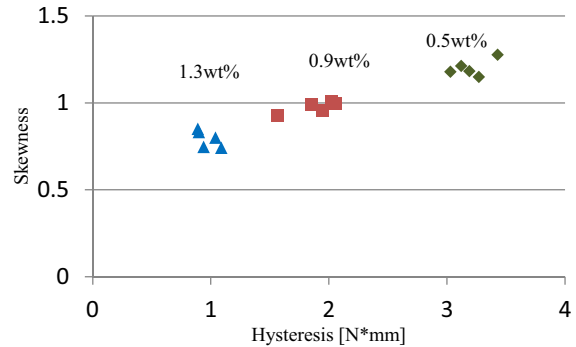


Fig.3. Skewness of normalized velocity map vs. hysteresis of stress-strain curve for carrageenan gel experiment.

### References

- [1] Y. Yamakoshi, J. Sato, and T. Sato: IEEE Trans. Ultrason. Ferroelect. Freq. Contr. **37**, 45 (1990)
- [2] T. Miwa, R.K. Parajuli, R. Tomizawa, and Y. Yamakoshi: Jpn. J. Appl. Phys. **50**, 07HF07 (2011)
- [3] R.K. Parajuli, R. Tei, D. Nakai, and Y. Yamakoshi: Jpn. J. Appl. Phys. **52** (2013) 07HF22.
- [4] H.T. Banks, G.A. Pinter: Multiscale Modeling & Simulation **3**(2), 395-412 (2005)
- [5] P.P. Delsanto, A.S. Gliozzi, M. Hirsekorn, M. Nobili: Ultrasonics **44**(3), 279-286 (2006)
- [6] I.P. Herman: Physics of Human Body, Springer, 2008, Chapter4.
- [7] S. Neumark, Techn. Sc. D., F.R.Ae.S: R.&M.No. 3269 (1957)
- [8] J.F. Greenleaf, M.Fatemi, and M. Insana: Annu. Rev. Biomed. Eng. **5**:57-78(2003)