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Classification of nonlinear resonant vibration modes by magnetic point groups

磁性点群による非線形共鳴振動モードの分類

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1. Introduction

Because of the technological and scientific importance, free-vibration acoustic resonance (FVAR) of elastic mediums has been investigated extensively since the era of L. Rayleigh and W. Ritz [1,2]. One of a seminal work has been conducted by E. Mochizuki [3], in which he classified linearized FVAR modes by irreducible representations of point groups. Compared to the linearized systems, however, FVAR of nonlinear elastic medium is still relatively little understood and further investigation is needed. Recently, one of the authors (R.T.) investigated FVAR of a two-dimensional hyperelastic material and revealed that (i) *colour symmetry* is embedded in the nonlinear FVAR modes and (ii) symmetry and structure of the nonlinear modes are expressed by magnetic point groups, rather than the irreducible representations [4]. By applying the result to a three-dimensional hyperelastic medium, we can generalize the linearized FVAR modes in [3] into finite amplitude nonlinear ones. However, to the authors' knowledge, classification of nonlinear FVAR modes has not been reported yet. In the present study, we investigate the symmetry of three-dimensional nonlinear FVAR modes for rectangular parallelepiped shape crystals and classify them on the basis of the magnetic point groups.

2. Magnetic Point Groups

2.1 Point groups of three-dimensional solids

Let us consider a rectangular parallelepiped shape nonlinear hyperelastic material which is defined by $\Omega = \{x_i | -L_i < x_i < L_i, i = 1,2,3\}$. To simplify the analysis, we suppose that the elastic symmetry of Ω is orthorhombic, monoclinic or triclinic. According to Mochizuki, the corresponding point group and symmetry operations are summarized as **Table 1** [3]. From the character table of group theory [5], we immediately see that the point group D_{2h} has eight types of irreducible representations denoted by A_g, B_{3g}, B_{2g}, B_{1g}, A_u, B_{3u}, B_{2u} and B_{1u}. Similarly, irreducible representations of the point group C_{2h} is A_g, B_g, A_u and B_u and those of the S_2 group is A_g and A_u.

Table 1. Point group symmetry and symmetry operations for rectangular parallelepiped shape triclinic, monoclinic and orthorhombic crystals.

Crystal system	Point group	Symmetry operators
Triclinic	S_2	$\{E, i\}$
Monoclinic	C_{2h}	$\{E, C_2, i, \sigma_h\}$
Orthorhombic	D_{2h}	$\{E, C_{2x}, C_{2y}, C_{2z}, i, \sigma_x, \sigma_y, \sigma_z\}$

2.2. Magnetic point group

Magnetic point group is consisting of symmetry operations in a point group and time reversal symmetry operation $\hat{T}$. Let $\mathcal{G}$ denotes a point group and let $\mathcal{H}$ be an invariant subgroup of index 2. Then, we can form a magnetic point group such that [5]

$$\mathcal{G}(\mathcal{H}) = \mathcal{H} + \hat{T}(\mathcal{G} - \mathcal{H})$$

This is called black and white, or *bicolour* magnetic point group. According to the expression, a conventional point group is written by $\mathcal{G} = \mathcal{G}(\mathcal{G})$ and is called black or white, or *single colour* magnetic point group. For the case of point group D_{2h} (orthorhombic crystal), there exists eight types of single and bicolour magnetic point groups: $D_{2h}(D_{2h})$, $D_{2h}(C_{2h}^x)$, $D_{2h}(C_{2h}^y)$, $D_{2h}(C_{2h}^z)$, $D_{2h}(C_{2v}^x)$, $D_{2h}(C_{2v}^y)$, $D_{2h}(C_{2v}^z)$ and $D_{2h}(D_2)$. Similarly, for point group C_{2h} (monoclinic crystal), we have four kinds of magnetic point groups: $C_{2h}(C_{2h})$, $C_{2h}(S_2)$, $C_{2h}(C_2)$ and $C_{2h}(C_{1h})$. For point group S_2, we have two magnetic point groups: $S_2(S_2)$ and $S_2(C_1)$.

2.2. Projection operation

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We next introduce the projection operator P for respective magnetic point groups. The definition of the operator is written in a standard textbook similar to [5]. For the case of magnetic point group $D_{2h}(C_{2h}^x)$ it becomes

$$P = \frac{(E + C_{2x} + \sigma_x + i) + \hat{T}(C_{2y} + C_{2z} + \sigma_y + \sigma_z)}{8}.$$

Let us express the displacement due to FVAR of Ω by

$$u_i = \sum_{p,q,r} \sum_n a_{n,p,q,r,i} \cos(n\omega t) P_p(x) P_q(y) P_r(z) \boldsymbol{e}_i.$$

Then, the equation $u_i = Pu_i$ project out a displacement component which is responsible for the vibration denote by $D_{2h}(C_{2h}^x)$.

3. Classification of Nonlinear FVAR modes

Tables 1 to 3 summarize the symmetry of displacement function obtained from the projection operations. In the tables, E (even) and O (odd) denote the parity of displacement function u_i for x_i direction. For $n = \mathrm{Odd}$ cases, the symmetry is identical to that of a linearized system. However, for $n = \mathrm{Odd}$ cases, it is identical to totally symmetric mode A_g. This result agrees well with previously obtained two-dimensional system.

Table 1. Classification of nonlinear FVAR modes for a rectangular parallelepiped shape orthorhombic crystal.

		n = Even			n = Odd		
group		p	q	r	p	q	r
$D_{2h}(D_{2h})$ A_g	u_1	O	E	E	O	E	E
	u_2	E	O	E	E	O	E
	u_3	E	E	O	E	E	O
$D_{2h}(C_{2h}^x)$ B_{3g}	u_1	O	E	E	O	O	O
	u_2	E	O	E	E	E	O
	u_3	E	E	O	E	O	E
$D_{2h}(C_{2h}^y)$ B_{2g}	u_1	O	E	E	E	E	O
	u_2	E	O	E	O	O	O
	u_3	E	E	O	O	E	E
$D_{2h}(C_{2h}^z)$ B_{1g}	u_1	O	E	E	E	O	E
	u_2	E	O	E	O	E	E
	u_3	E	E	O	O	O	O
$D_{2h}(D_2)$ A_u	u_1	O	E	E	E	O	O
	u_2	E	O	E	O	E	O
	u_3	E	E	O	O	O	E
$D_{2h}(C_{2v}^x)$ B_{3u}	u_1	O	E	E	E	E	E
	u_2	E	O	E	O	O	E
	u_3	E	E	O	O	E	O
$D_{2h}(C_{2v}^y)$ B_{2u}	u_1	O	E	E	O	O	E
	u_2	E	O	E	E	E	E
	u_3	E	E	O	E	O	O
$D_{2h}(C_{2v}^z)$ B_{1u}	u_1	O	E	E	O	E	O
	u_2	E	O	E	E	O	O
	u_3	E	E	O	E	E	E

Table 2. Classification of nonlinear FVAR mode for a rectangular parallelepiped shape monoclinic crystal.

		n = Even		n = Odd	
group		$p+q$	r	$p+q$	r
$C_{2h}(C_{2h})$ A_g	u_1	O	E	O	E
	u_2	O	E	O	E
	u_3	E	O	E	O
$C_{2h}(S_2)$ B_g	u_1	O	E	E	O
	u_2	O	E	E	O
	u_3	E	O	O	E
$C_{2h}(C_2)$ A_u	u_1	O	E	O	O
	u_2	O	E	O	O
	u_3	E	O	E	E
$C_{2h}(C_{1h})$ B_u	u_1	O	E	E	E
	u_2	O	E	O	O
	u_3	E	O	O	O

Table 3. Classification of nonlinear FVAR mode for a rectangular parallelepiped shape triclinic crystal.

		n = Even	n = Odd
group		$p+q+\mathrm{r}$	$p+q+r$
$S_2(S_2)$ A_g	u_1	O	O
	u_2	O	O
	u_3	O	O
$S_2(C_1)$ A_u	u_1	O	E
	u_2	O	E
	u_3	O	E

4. Conclusions

In this study, we investigated the symmetry of finite amplitude nonlinear FVAR modes for rectangular parallelepiped shape orthorhombic, monoclinic and triclinic crystals. Projection operations, derived from magnetic point groups, revealed that the symmetry depends on the parity of n: for $n = \mathrm{Odd}$ cases, it is identical to that of a linearized system whereas otherwise it becomes A_g. This result agree well with that obtained from a two-dimensional system.

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