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Equivalent Network Representation for a Backward-Wave-Type Trapped-Energy Resonator with Circular Electrodes
円形電極を持つ周波数上昇型エネルギー閉じ込め共振子の分布定数等価回路表示について

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1. Introduction

One of the authors presented a novel technique of energy trapping which was applied for thickness vibrations of backward-wave modes[1,2]. In this technique, special arrangement of the electrodes and the introduction of a capacitance connected in series with the excitation electrodes enabled us to create trapped-energy vibrations. Special features of this energy trapping were clarified by the equivalent-network analysis[2]. However, the equivalent-network model employed at that time was for a two-dimensional analysis and not for resonators having circular electrodes. In this paper, an equivalent network representation is presented for a backward-wave-type trapped-energy resonator having circular electrodes, basing on the network representation presented by Nakamura *et al.*[3] Distinguishing features of the trapped-energy modes of backward-wave type are reconfirmed.

2. Equivalent network in cylindrical coordinates

In some thickness vibration modes, the dispersion relation between the angular frequency ω and the wavenumber γ along the plate around the cut-off frequency has a form as shown in **Fig. 1(a)**. In this case, the corresponding vibration becomes a backward-wave mode. To realize energy trapping, the surrounding region should be electroded and short-circuited such as shown in **Fig. 1(b)** [1,2].

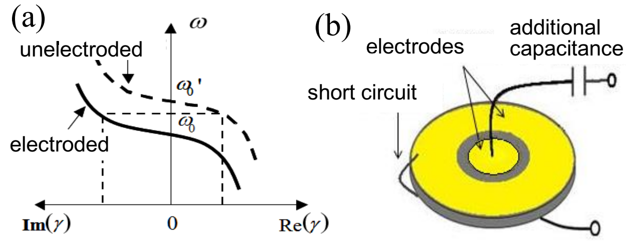

Fig.1 (a) Dispersion curve for a backward-wave-mode thickness vibration and (b) electrode configuration for energy trapping.

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Nakamura *et al.* [3] presented an equivalent network representation for a conventional trapped-energy resonator with circular electrodes. The basic component of the network represents the radial propagation of an axisymmetric thickness-vibration mode in a ring-shaped electroded section having mechanical ports defined at its inner and outer cross sections.

A backward-wave-type trapped-energy resonator with circular electrodes and its equivalent network representation are shown in **Figs. 2** and **3**, respectively. The thickness of the piezoelectric ceramic plate is *2H*, the diameter of the central electrodes is *2a*, and the inner diameter of the outer electrodes is 2*b*. The network consists of three parts each representing the central electroded part, the un-electroded gap, and the surrounding electroded part. Addition of a series capacitance C_A is also taken into consideration.

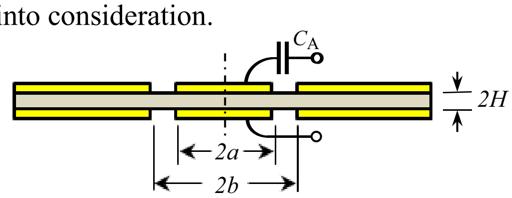
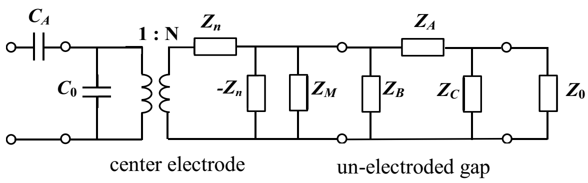


Fig. 2 Trapped-energy resonator with circular electrodes.



$$N = \frac{\pi a^2 e_{33}}{H} \qquad C_0 = \frac{\pi a^2 \varepsilon_{33}}{2H}$$

$$Z_n = j\frac{\pi^2 a^2 H c_{44} \gamma^2}{\omega} \qquad Z_M = -\frac{2\pi a H c_{44}\gamma}{\omega}\cdot\frac{J_1(\gamma a)}{J_0(\gamma a)}$$

$$Z_A = -\frac{\pi^2 abHc_{44}\gamma'^2}{j\omega}\cdot\{J_1(\gamma' a)\cdot Y_1(\gamma' b) - J_1(\gamma' b)\cdot Y_1(\gamma' a)\}$$

$$Z_B = \frac{2\pi a H c_{44}\gamma'}{j\omega}\cdot\frac{J_1(\gamma' a)\cdot Y_1(\gamma' b) - J_1(\gamma' b)\cdot Y_1(\gamma' a)}{J_0(\gamma' a)\cdot Y_1(\gamma' b) - J_1(\gamma' b)\cdot Y_0(\gamma' a) + \frac{2}{\pi\gamma' b}}$$

$$Z_C = \frac{2\pi b H c_{44}\gamma'}{j\omega}\cdot\frac{J_1(\gamma' a)\cdot Y_1(\gamma' b) - J_1(\gamma' b)\cdot Y_1(\gamma' a)}{J_0(\gamma' b)\cdot Y_1(\gamma' a) - J_1(\gamma' a)\cdot Y_0(\gamma' b) + \frac{2}{\pi\gamma' a}}$$

$$Z_0 = \frac{2\pi b H c_{44}\gamma}{j\omega}\cdot\frac{J_0(\gamma b)\cdot J_1(\gamma b) + Y_0(\gamma b)\cdot Y_1(\gamma b) + j\frac{2}{\pi\gamma b}}{\{J_0(\gamma b)\}^2 + \{Y_0(\gamma b)\}^2}$$

Fig.3 Equivalent network and constants of the elements.

The constants of the network elements are given by combinations of Bessel and/or Hankel functions. The wavenumbers γ and γ' in the arguments are determined for a given ω from the dispersion relations.

3. Results of the analysis

3.1 Admittance characteristics

Admittance characteristics without and with an additional capacitance C_A computed using the network are shown in **Figs. 4** and **5**, respectively. Here, the vertical axis is the normalized admittance $Y/(v_l C_0/H)$ and the horizontal axis is the normalized frequency $\omega H/v_l$ (v_l: longitudinal wave velocity). A thickness-poled $PbTiO_3$ plate is assumed as the piezoelectric material, and the geometries of the electrodes are assumed as a/H=4, $(b-a)/2H$=0.5. In Fig. 4, trapping of the vibration energy is not sufficient and spurious response due to the energy leakage in the lower frequency region is noted. Relatively clean single-resonance characteristic is realized in Fig. 5 where C_0/C_A is assumed to be 1.

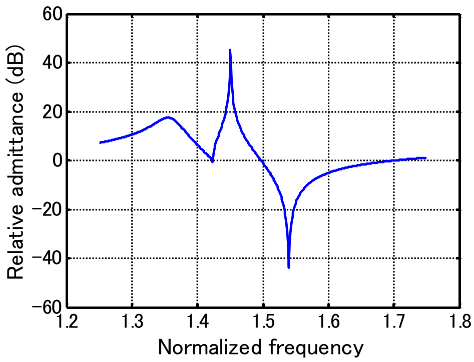


Fig. 4 Admittance characteristic without C_A

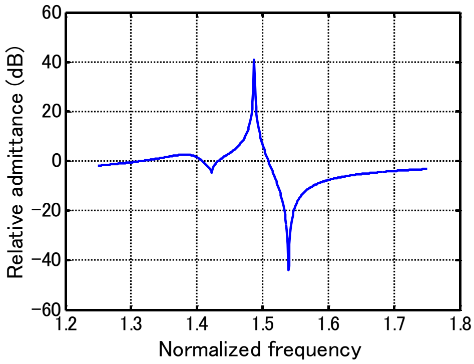


Fig. 5 Admittance characteristic when C_0/C_A=1.

3.2 Frequency spectra

It has been shown in the former report[2] that resonance and anti-resonance frequency spectra of the backward-wave-type tapped-energy mode vary depending mainly on the size of the un-electroded gap between the center and the outer electrodes. In order to reconfirm this fact, resonance (f_R) and anti-resonance (f_A) frequency spectra are computed. **Figure 6** shows the variations of f_R and f_A with the normalized gap width $(b-a)/2H$ obtained for a/H=1. The vertical axis is the normalized frequency $(\omega-\omega_0)/(\omega_0'-\omega_0)$ ranging in between the upper cutoff frequency ω_0' for the un-electroded plate and the lower cutoff frequency ω_0 for the electrode plate. It is noted that many inharmonic undertone modes appear for the large gap width. **Figure 7** shows the spectra when the ratio $(b-a)/2H$ is fixed to 2 and a/H is varied. The number of trapped-energy modes does not depend on the size of the central electrodes.

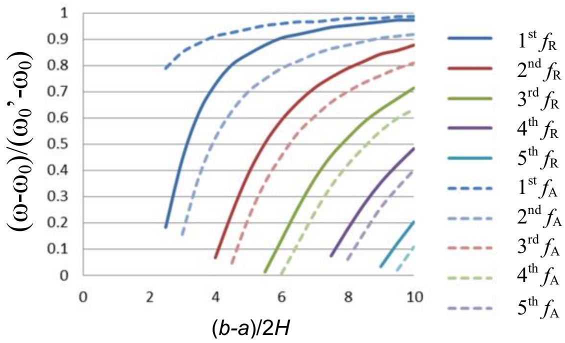


Fig. 6 Frequency spectra for variation in $(b-a)/2H$ (a/H=1)

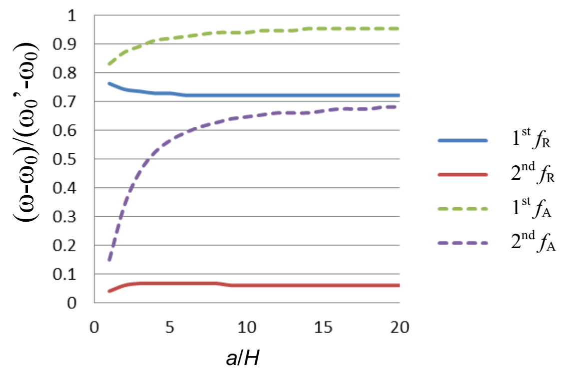


Fig. 7 Frequency spectra for variation in a/H ($(b-a)/2H$=2)

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