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Numerical Simulations of a Thermoacoustic Prime Mover at 20 kHz

20 kHz における熱音響の数値シミュレーション

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1. Introduction

Miniaturization of thermoacoutsic system is required for some practical applications of the system such as for the usage of waste heat from some electrical circuits. Some research gropus have already performed experiments using a high frequency sound such as in the kHz range in order to miniaturize the system because the resonator (pipe) length is smaller for higher sound frequency.

Flitcroft and Symko[1] made a thermoacoustic prime mover (sound generator) which operates at about 20 kHz with the resonator length of about 3.4 mm. The working fluid was air at atmospheric pressure. A stack in which heat is transformed into sound energy was made of random stainless-steel mesh instead of a set of straight narrow tubes. By the temperature difference of about 300 °C across the prime mover, a sound wave of about 20 kHz and 300 Pa in frequency and pressure amplitude, respectively was generated in the system.

In the present study, numerical simulations of a thermoacoustic prime mover at 20 kHz are performed by the trasnfer matrix method of Ueda[2] which is based on the Rott's acoustic approximation.

2. Model

In the present study, a stack is assumed to consist of a set of narrow straight tubes. Furthermore, it has been suggested that the capillary-tube-based thermoacoustic theory is also valuable in predicting the acoustic properties of random porous media such as a stack of aluminum foam[3]. The momentum and continuity equations for a tube are given by Eqs. (1) and (2)[2].

$$dP/dx = -(i\omega\rho\_m)/(1-\chi\_\nu\ )\ U \quad (1)$$

$$dU/dx = -i\omega[1+(\gamma-1)\ \chi\_\alpha\ ]/(\gamma P\_m\ )\ P + (\chi\_\alpha - \chi\_\nu)/(1-\chi\_\nu\ )(1-\sigma)\ \ 1/T\_m\ \ (dT\_m)/dx\ U \quad (2)$$

where P and U are the sound pressure and the particle velocity, respectively which are complex numbers including the information of their phases, x is the position along the straight-tube with its origin at the lower temperature side of a stack, i is the unit imaginary number, ω is the angular frequency of an acoustic wave, ρ_m, P_m, γ, σ are the mean density, the mean pressure, the ratio of specific heats, and the Prandtl number of the working gas, respectively. χ_α and χ_ν are the thermoacoustic functions given in Refs. 2.

In the transfer matrix method of Ueda[2], Eqs. (1) and (2) are expressed using a matrix as Eq. (3).

$$(P_{n+1}, U_{n+1}) - (P_n, U_n) = M_{b,n}(P_n, U_n) \quad (3)$$

where a narrow tube is divided into N short tubes, P_n and U_n are the sound pressure and particle velocity at the nth short tube (n=0 corresponds to the lower temperature side of a tube (stack)), and $M_{b,n}$ is a matrix defined in Ref. 2 . Using Eq. (3), the sound pressure and the particle velocity at any point in a stack are calculated from those at the lower temperature side of a stack (n=0 or x=0).

$$\big(P(x_n), U(x_n)\big) = \big(E + M_{b,n-1}\big)\big(E + M_{b,n-2}\big) \dots (E + M_{b,1})(E + M_{b,0})(P(x_0), U(x_0)) \quad (4)$$

where E is the unit matrix.

When there is negligible thermal conduction between a tube and the surroundings, the total enthalpy flux (H_{flux}) is conserved along a narrow tube. The work flow (acoustic power) (I) is calculated by Eq. (5).

$$I = A|P||U|\cos(\phi - \theta)/2 \quad (5)$$

where A is the area of the cross section of a stack where the working gas moves, ϕ and θ are the phase angle of the acoustic pressure (P) and that of the particle velocity (U), respectively. The heat flux is calculated by Eq. (6).

$$Q = H_{flux} - I \quad (6)$$

Finally, two alternate definitions of the energy efficiency are discussed. The first definition is the rigorous physical definition as follows[2].

$$\eta_{phy} = \Delta I/Q_H \quad (7)$$

where η_{phy} is the energy efficiency, ΔI is the change in the work flow across a stack ($\Delta I = I_H - I_C$ where I_H and I_C is the work flow at the higher and lower temperature side, respectively), and Q_H is the heat flux at the higher temperature side of a

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stack. The other definition (η_{eng}) is useful from the viewpoint of engineering[4].

$$\eta_{eng} = \bar{I}/\bar{Q} \qquad (8)$$

where $\bar{I}$ and $\bar{Q}$ is the spatial average of the work flow in stack and that of the heat flux, respectively.

3. Results and Discussions

In the present study, numerical simulations are performed for 20 kHz in sound frequency. The radius of a narrow tube in a stack is 0.02 mm. The other conditions are the same as those in the experiment of Ueda et al.[5] The working gas is air at atmospheric pressure. The pressure amplitude of sound at x=0 is 3.4 kPa. For the work flow to increase, the total enthalpy flux should be larger than $30I_C$ in magnitude. In Fig. 1, the result for H_{flux}=-200Ic is shown. There is a critical length in a stack above which the work flow begins to decrease. The reason of the decrease is the decrease of cos(ϕ-θ) due to the increase in ϕ-θ with distance in a stack.

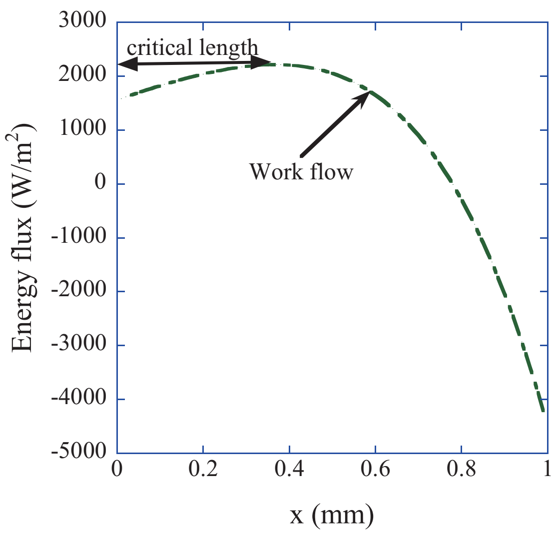

Fig. 1 The result of the numerical simulation on the work flow as a function of x in a stack.

In Fig. 2, the results of the numerical simulations for various radii of a narrow tube on the energy efficiency defined are shown. The engineering efficiency (Eq. (8)) is always higher than the physical one (Eq. (7)). The physical one is as low as 0.1-0.5 % even at the optimum $\omega\tau_\alpha$ where ω is the angular frequency of sound and τ_α is the thermal relaxation time at the lower temperature side of a stack (x=0)[2].

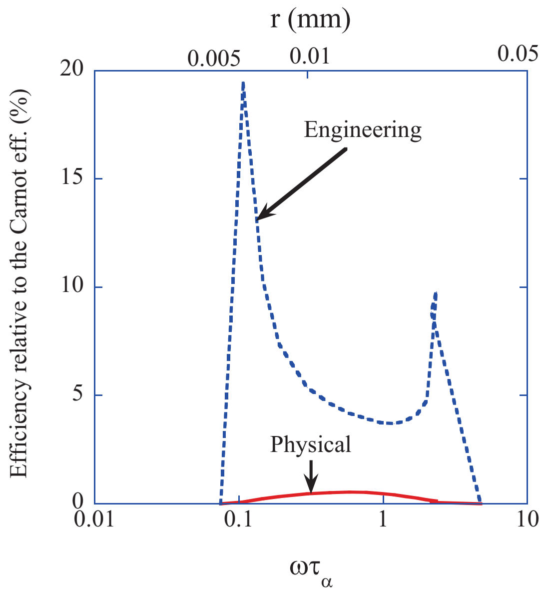

Fig. 2 The results of the numerical simulations on the energy efficiency

4. Conclusions

Numerical simulations of a thermoacoustic prime mover at 20 kHz have revealed that there is a critical length in a stack above which the work flow begins to decrease. The reason is the increase in the difference of phase angles between pressure and particle velocity of sound in a stack with distance. Two alternate definitions of energy efficiency are discussed.

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