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Effects of Ignoring Mean Normal Stress Distribution on Shear Modulus Reconstruction

平均垂直応力分布を無視することによるずり弾性率再構成への影響

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1. Introduction

Mechanical properties such as viscoelasticities (e.g., [1]) are estimated using various mechanical sources such as a heart motion, a low frequency compression/stretching, an applied vibration, and an acoustically radiated force etc. Such estimation is performed through measurements of deformations (strains etc.), a speed of shear wave propagations etc. Such mechanical properties can be reconstructed numerically or via signal processing. A stress tensor, internal mechanical sources and a mean normal stress can also be reconstructed simultaneously. However, various artifacts possibly occur under various assumptions such as an incompressibility of tissues, a low dimensionality of mechanical property distributions (e.g., [2-6]), etc.

For the reconstruction of a shear modulus distribution, the distribution of a mean normal stress is sometimes ignored. In this report, the effects of ignorance are investigated through simulations. Theoretically, an assumption of a uniform mean normal stress distribution leads to reconstruction errors.

2. Ignoring mean normal stress

For the multidimensional reconstruction of a shear modulus, Methods A to C [4,5] use the mean normal stress as an unknown, whereas Method F [3] uses a typical Poisson ratio in governing equations, i.e., for Methods A to C,

$$\rho\alpha_i = p_{,j}\delta_{ij} + 2\varepsilon_{ij}G_{,j} + 2\varepsilon_{ij,j}G \qquad (1)$$

and for Method F,

$$\rho\alpha_i = \{\varepsilon_{\alpha\alpha}\delta_{ij}\}\{2\nu/(1-2\nu)\}G_{,j} + \{\varepsilon_{\alpha\alpha}\delta_{ij}\}_{,j}\{2\nu/(1-2\nu)\}G + 2\varepsilon_{ij}G_{,j} + 2\varepsilon_{ij,j}G, \qquad (2)$$

where G is the target shear modulus, p is the mean normal stress, ν is the Poisson ratio, ε_{ij} is the measured strain tensor, ρ is the density, α_i is the measured acceleration vector, and δ_{ij} is *a delta function*.

Method B [4] permits the simultaneous reconstruction of a mean normal stress p by using a reference mean normal stress or a reference Poisson's ratio ν with a measured volume strain $\varepsilon_{\alpha\alpha}$; and Methods A [4] and C [5] permit a shear modulus reconstruction with remaining the mean normal stress unknown by using no reference mean normal stress and by using the quasi-reference mean normal stress (*e.g.*, zero) instead of an absolute mean normal stress, respectively. However, for the mean normal stress reconstruction, Method C yields an absolute difference distribution dependent on the used quasi-reference mean normal stress and by using an iterative solution (*e.g.*, a conjugate gradient method), Method A yields a relative distribution dependent on an initial estimate (*e.g.*, unity) used for the iterative solution. Because a relative (Method E using a quasi-reference shear modulus such as a unity) or absolute shear modulus reconstruction can be performed with a high accuracy, and no geometrical artifacts are also generated in the mean normal stress reconstructions, both of the quasi-reconstructions are useful. Moreover, being different from Method F except for a 2D reconstruction using the assumption of a 2D stress condition [3] (i.e., a 2D reconstruction different from using the assumption of a 2D strain condition [3]), Methods A to C permit dealing with completely incompressible tissues ($\nu \approx 0.5$) [4,5]. Moreover, an inhomogeneous Poisson's ratio can also be dealt with [4,5]. The uses of a typical density value (1.0×10^3 kg/m^3) is also effective for decreasing an unknown variable and increasing the speed of a calculation [3-5].

When ignoring the spatial distribution of p, the last two terms remain in the right-hand-side. Originally, the reasons why p was ignored are that p becomes indefinite theoretically and p cannot also be measured accurately for incompressible tissues because the volume strain $\varepsilon_{\alpha\alpha}$ is infinitesimal and Poisson's ratio nearly equals to 0.5 [see eq. (2)]. Recall that when using Method F, if an error exists in the value used for the Poisson's ratio ν, serious errors occur in reconstructed shear modulus values [3]. In contrast, Methods A to C do not yield any such reconstruction errors.

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3. Simulations

Various linear numerical cubic phantoms (50 mm sides) were dealt with (**Fig. 1**). For instance, a Poisson's ratio was assumed to be uniform or nonuniform (~0.49). The phantoms had a stiff or soft spherical inclusion (10 mm dia.). The respective phantoms were compressed/stretched or vibrated in a depth direction using large external sources generated at the upper planes of the phantoms. The forward calculation was performed using the successive-over-relaxation (SOR) method. Using the reconstruction Method B, the shear modulus distribution was reconstructed together with the mean normal stress distribution; and the shear modulus was also reconstructed using the method ignoring a mean normal stress distribution. The means and standard deviations (SDs) of reconstructed shear moduli were estimated in inclusions.

3. Results

For both phantoms having uniform Poisson's ratios 0.48 and 0.47 and an inclusion with a shear modulus higher than the surrounding region (2.0 vs 1.0×10^5 N/m^2), as theoretically predicted, the shear moduli of the inclusion was estimated to be **larger** than the original value, i.e., inaccurate [for instance, when Poisson's ratio = 0.48, means (SDs) were 2.07 (0.16) vs 2.02 (0.07)$\times10^5$ N/m^2]. The SDs also became larger (i.e., **about twofold SDs and unstable**) [see **Table I**]. See also log-gray-scaled images shown in **Fig. 2**. At the surrounding regions of the inclusions, reconstruction errors were detected rather for 0.48 than for 0.47.

For soft inclusion phantoms, as theoretically predicted, the shear modulus was estimated to be **significantly smaller** than the original value (for instance, for a half shear modulus, 0.5×10^5 N/m^2 and Poisson's ratio, 0.47, means and SDs were respectively **0.19 (0.01) vs 0.52 (0.02)$\times$10^5 N/m^2** (**Table I**).

For instance, when the twofold shear modulus inclusion had a remarkedly smaller Poisson's ratio than the surrounding (e.g., smaller than 0.43 vs 0.47), the shear modulus was estimated to be smaller than that of the surrounding (e.g., for 0.42, the mean was 0.73×10^5 N/m^2).

For the respective same phantoms, completely the same results were obtained in compression and stretching cases.

4. Conclusions

The effects of ignoring a mean normal stress distribution were investigated. Theoretically predictable results were also numerically obtained.

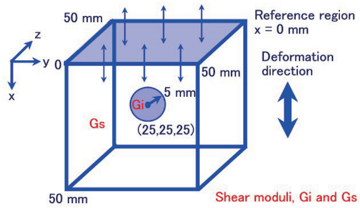

Fig. 1 Simulated phantom. Shear moduli, Gi = 2×10^5 N/m^2, Gs = 1×10^5 N/m^2.

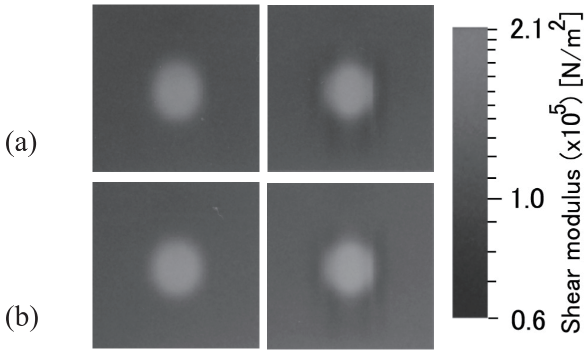

Fig. 2. Log-gray-scaled images of shear modulus reconstructions using Method B (left) and ignoring mean normal stress (right) for (a) Poisson's ratios = 0.48 and (b) 0.47.

Table I. Means and SDs of reconstructed shear moduli Gi and Poisson's ratios *v* in stiff and soft inclusions.

Gi($\times10^5$N/m^2)	2.00		0.50
v	0.48	0.47	0.47
Method B with *v*	2.02 (0.07) 0.480(0.001)	1.99 (0.06) 0.470(0.001)	0.52 (0.02) 0.469 (0.001)
Ignoring p	2.07 (0.16)	2.07 (0.15)	0.19 (0.01)

Similar artifacts are generated when ignoring internal mechanical sources, viscosity, nonlinear properties, isotropic properties. Occasionally performed assumption of a local homogeneity also affects the reconstruction severely (i.e., decrease in a spatial resolution as well). At the symposium, the limitations caused by such assumptions will also be referred to together with results obtained in a low frequency oscillation case.

References

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