

**Measurement of Shear Modules by Continuous-Wave Ultrasonic Actuation Using Dual Acoustic Radiation Pressure**

音響放射圧を用いた双方向局所連続加振による  
 剪断弾性率計測

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**1. Introduction**

Although it is important to diagnose physical conditions of muscle and tendon, quantitative and noninvasive diagnosis method has not yet been established. In our previous study, we developed a method, which uses ultrasonic acoustic radiation pressures irradiated from two opposite horizontal directions<sup>1)</sup>. This method enables effective generation of strain by continuous-wave ultrasonic actuation.

In the present study, we estimated shear elasticity and viscosity of the phantom from one-dimensional Helmholtz equation for Voigt model using the measured propagation velocity of shear wave. We evaluated the estimated shear modulus of a phantom, which simulates a soft biological tissue.

**2. Principle**

**2.1 Acoustic radiation pressure at frequency difference of two continuous ultrasounds**

When the ultrasound propagates in a medium, a constant force is generated in the direction of propagation. This force is called as the acoustic radiation force. The acoustic radiation pressure is defined as the acoustic radiation force per unit area<sup>2)</sup>. When two ultrasounds with the same sound pressure,  $p_0$ , at slightly different frequencies,  $f_0$  and  $(f_0 + \Delta f)$ , are crossed each other, an acoustic radiation pressure which fluctuates at the frequency difference,  $\Delta f$ , is generated in the intersectional space. In present study, we use this fluctuating force. The sound pressure,  $p_{sum}(t)$ , generated in the intersectional space is given by:

$$p_{sum}(z, t) = e^{-\alpha z} [p_0 \cos(kz - 2\pi f_0 t) + p_0 \cos\{kz - 2\pi(f_0 + \Delta f)t\}], \quad (1)$$

where  $z$ ,  $\alpha$  and  $k$  are the depth, attenuation coefficient and wavenumber of vibration of the object, respectively.

Since displacement of the object generated by the high frequency components are negligible compared to displacements generated by the low

frequency components including the direct current component and  $\Delta f$ . The acoustic radiation pressure,  $P_R(z, t)$ , is approximately given by

$$P_R(z, t) \approx \frac{\alpha p_0^2}{2\rho_2 c_2^2} e^{-2\alpha z} (1 + 2 \cos 2\pi \Delta f t), \quad (2)$$

where  $\rho_2$  and  $c_2$  are the density and sound speed in the object, respectively.

**2.2 Estimation of shear modulus from propagation velocity of shear wave**

In order to generate strain efficiently and observe the propagation of shear wave within the object, Odagiri *et al.* used two acoustic radiation pressures<sup>1)</sup>. When two radiation pressures are synchronously applied to the object as shown in Fig. 1, shear wave propagates dominantly in the  $x$ - $y$  plane. The shear modulus,  $G$ , is estimated from the measured shear wave velocity,  $c_s$ , as follows:

$$G = \rho c_s^2 \quad (3)$$

where  $\rho$  is the density of the object. The velocity of shear wave propagation,  $c_s$ , is experimentally determined as follows:

$$\hat{c}_s = 2\pi \Delta f \cdot \Delta l / \Delta \theta, \quad (4)$$

where  $\Delta l$  and  $\Delta \theta$  are the intervals of ultrasonic beams for measurement and the phase difference between displacements of the object measured in neighboring beams, respectively.

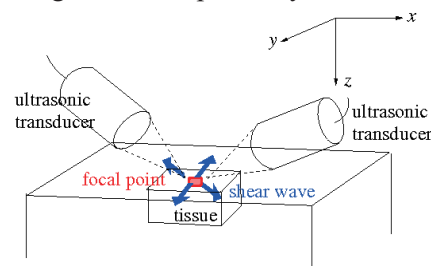


Fig. 1. Schematic diagram of the shear wave propagation.

**2.3 Experimental method and theoretical evaluation with Voigt model**

Figure 2 shows a schematic diagram of the experimental system. Silicone rubber, which simulates a soft biological tissue, were measured to observe the propagation of the shear wave. The hardness of the silicone rubber was expressed by the ASKER durometer type C of 0. The propagation velocity of shear wave was estimated by applying

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the least squares fitting to the mean values of the phase spectra of displacements obtained for three measurements.

On the other hand, using one-dimensional Helmholtz equation for Voigt model, the shear wave velocity  $c_s$  and attenuation  $\alpha_s$  are modeled as<sup>3)</sup>:

$$c_s = \sqrt{\frac{2(\mu_1^2 + \omega^2 \mu_2^2)}{\rho(\mu_1 + \sqrt{\mu_1^2 + \omega^2 \mu_2^2})}}, \quad \alpha_s = \sqrt{\frac{\rho \omega^2 (\sqrt{\mu_1^2 + \omega^2 \mu_2^2} - \mu_1)}{2(\mu_1^2 + \omega^2 \mu_2^2)}} \quad (5)$$

where  $\mu_1$ ,  $\mu_2$ ,  $\rho$ , and  $\omega$  are the shear elasticity, shear viscosity, density of propagation media, and angular frequency, respectively. In the present study, we used only shear wave velocity since attenuation of shear wave is sensitive to noise<sup>3)</sup>.

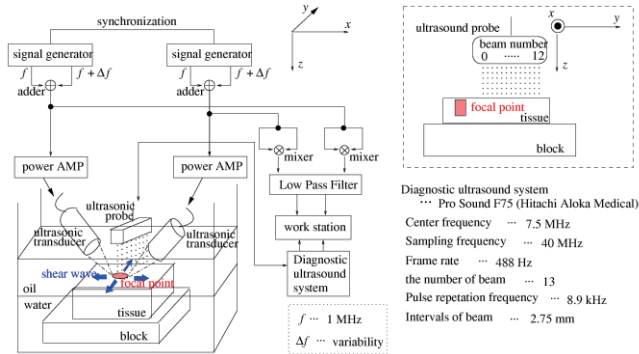


Fig. 2 Schematic diagram of the experimental method.

### 3. Results

Figure 3 shows the displacement waveforms at the position of 5 mm in depth of silicone rubber at  $\Delta f = 5$  Hz. Their normalized amplitudes of displacement are shown in Fig. 4(a), and the phase spectra at 5 Hz is shown in Fig. 4(b). Plots and vertical bars show means and the minimum to maximum values. As shown in Fig. 4(a), the displacement caused by ultrasonic actuation is attenuated during propagation. The mean of normalized displacement amplitude was well expressed by an exponential function. Shear wave velocity was estimated to be 0.72 m/s from the estimated gradient of phase spectra of 0.044 rad/mm, as shown in Fig. 4(b).

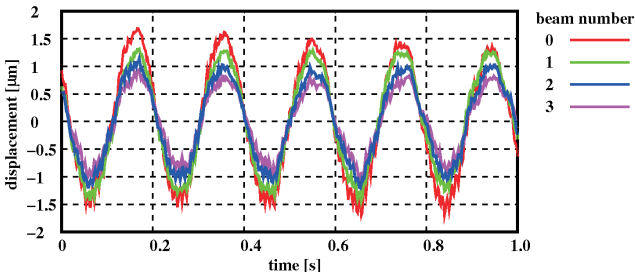


Fig. 3 The displacement waveforms of silicone rubber.

The propagation velocity of shear wave is plotted as a function of  $\Delta f$  as shown in Fig. 5. Plots and vertical bars show means and the minimum to maximum values. The mean of phase spectra well fit to the model in Eq. (5). The estimated elasticity

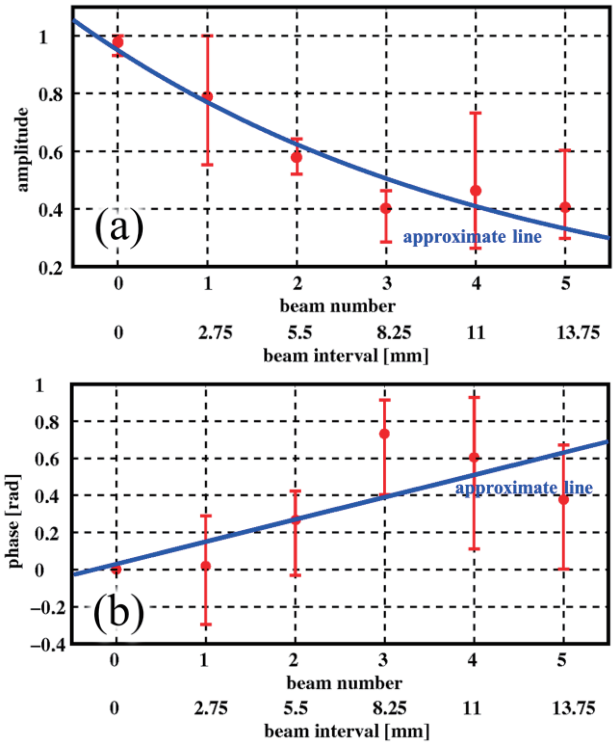


Fig. 4 The position of 5 mm in depth of silicone rubber (a) the normalized displacement amplitude, (b) the phase spectra at 5 Hz.

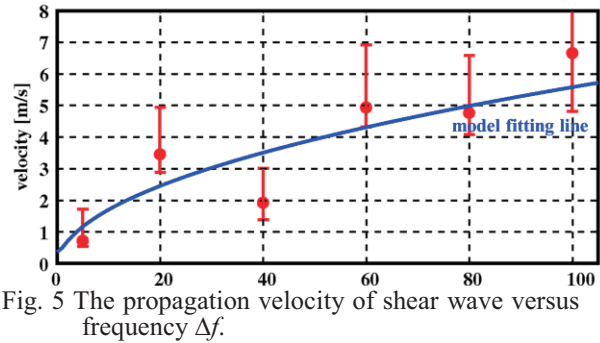


Fig. 5 The propagation velocity of shear wave versus frequency  $\Delta f$ .

and viscosity of the silicone rubber were  $\mu_1 = 1.0$  kPa, and  $\mu_2 = 6.2$  Pa s, respectively.

### 4. Conclusion

In this study, we estimated viscoelastic properties of silicone rubber, which simulates a soft biological tissue, using the acoustic radiation pressure. The results show a possibility of the proposed method for noninvasive estimation of the regional properties of an human organ by measuring velocity of the shear wave effectively generated by acoustic radiation pressure using low-intensity continuous wave ultrasound.

### References

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