

Ripplon Resonance in Confined Liquid Surface

リップロン共鳴現象の観察

Toshiyuki Koga^{1‡}, Shujiro Mitani¹, and Keiji Sakai¹

(¹Institute of Industrial Science, the University of Tokyo)

古賀 俊行^{1‡}, 美谷 周二朗¹, 酒井 啓司¹ (¹東京大学生産技術研究所)

1. Introduction

Ripplon is a thermally fluctuated capillary wave, whose propagation can be observed with a ripplon light scattering technique in a non-contact and non-destructive manner. We applied this technique for the dynamic molecular properties at the free liquid surface or liquid-liquid interface. In this presentation, we report the ripplon propagation on a confined liquid surface. In the confined liquid surface, the ripplon spectrum is expected to split due to the resonance of the ripplon. We show experimentally the resonant spectrum of ripplon on the pure water and 0.65 cSt silicone oil surfaces, whose areas are restricted by boundaries.

2. Theory

The propagation of the ripplon is determined by the surface tension σ , the density of the liquid ρ , and the bulk viscosity of the liquid η . In our experiment, the angular frequency ω_0 and the temporal damping constant Γ of the ripplon can be approximated as:

$$\omega_0 = \sqrt{\frac{\sigma k^3}{\rho}}, \quad (1)$$

$$\Gamma = \frac{2\eta k^2}{\rho}. \quad (2)$$

The details of the ripplon spectroscopy technique was reported^[1], and we will briefly explain the principle. The laser light incident to the liquid surface is scattered by riplons, acting as weak phase gratings to the light. The wavenumber of the observed ripplon is determined by the scattered angle θ , which follows the Bragg condition, describing the momentum conservation:

$$k = K \sin \theta,$$

where k and K are the wavenumbers of the observed ripplon and the incident light, respectively. Ripplon propagates with a constant phase velocity, and the frequency of the incident light is modulated through the scattering process by the Doppler effect describing the energy conservation:

$$\omega_s = \omega_i \pm \omega_0,$$

where ω_s , ω_i , and ω_0 are the frequencies of the scattered light, incident light and the observed ripplon, respectively. The frequency of the scattered light is shifted up by the ripplon propagating in one direction, and down by one propagating in the opposite direction. Analyzing the scattered light with the optical heterodyne technique, we can obtain the ripplon power spectrum. This spectrum consists of two components. Each component is known to be approximated by the Lorentz curve, which has a sharp peak at $\omega = \pm\omega_0$ and a half width at the half maximum Γ . The above discussion holds for the ripplon propagating on a free surface.

On the other hand, if a ripplon is confined in a cavity smaller than the spatial damping length of the ripplon, the same ripplon intersects the laser beam after a round trip between the two boundaries. As a result, only the riplons whose phase matches that after the round trip, make intense light scattering signals. The condition of the resonance mode is then expressed as^[2]:

$$k_n L = n\pi, \quad (3)$$

where n is an integer, L is a width of the resonance cavity, and k_n is the wavenumber of the resonant ripplon. The observed phase is shifted by $2kL$ after each round trip. Taking this into consideration, the power spectrum of the ripplon can be calculated as^[3]:

$$S(k, \omega) \propto \left[\frac{\Gamma}{(\omega - \omega_0)^2 + \Gamma^2} + \frac{\Gamma}{(\omega + \omega_0)^2 + \Gamma^2} \right] \times \frac{1}{1 - 2e^{-2\alpha L} \cos(2kL) + e^{-4\alpha L}}, \quad (4)$$

where α is the spatial damping constant defined as $\alpha = 4\omega\eta/3\sigma$. Eq. (4) is the product of the normal ripplon doublet, and the resonance condition. The exact form of the ripplon resonance spectrum depends on the width L and the surface tension σ .

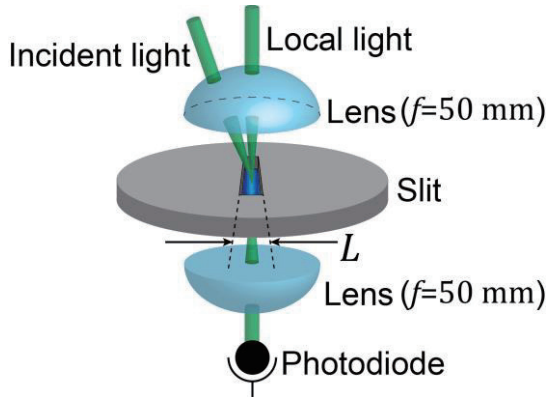


Fig. 1 Schematic view of the sample cell and the ripplon light scattering system.

3. Observation of ripplon resonance

Figure 1 shows the schematic view of the sample cell and the ripplon light scattering technique. The laser beam with an initial diameter of 2.34 mm is focused by a lens with a focal length of 50 mm. The angular divergence of the beams induced by the lens gives a wider detection band of ripples, which corresponds to the instrumental band. In our experiment, the slit is used as a resonant cavity, which is fixed on the sample liquid surface. The local light and the incident light intersect in the center of the cavity.

Figure 2 shows the ripplon power spectrum obtained for distilled water surface in the 100 μm -cavity at the scattered angle of 0.28° . The spatial damping length of ripplon due to viscosity is 1.5 mm, and therefore, the ripplon propagates back and forth within the cavity for about 8 times. The resonant orders of each peak are identified with the surface tension of distilled water and denoted by $n = 2, 3$, and 4, which are shown by arrows in **Fig. 2**.

Figure 3 shows the ripplon power spectrum obtained for 0.65 cSt silicone oil surface in the cavity with the width of 200 μm at the scattered angle of 0.78° . The spatial damping length of ripplon due to viscosity is 750 μm , and therefore, the ripplon propagates back and forth within the cavity for about 4 times. The surface tension of the sample is 15.6 mN/m. We can clearly see many resonant peaks, where the order is from $n = 4$ to 11. From Eqs. (1) and (3), we find that the interval of the resonant peak frequency becomes much shorter, as the width of the cavity increases and the surface tension becomes smaller. Hence, we can see more resonant peaks in **Fig. 3** than in **Fig. 2**.

The dashed curves in **Fig. 2** and **Fig. 3** show the spectrum observed on the free surface of the same liquid at the same scattered angle without any boundaries. These spectra approximately give the

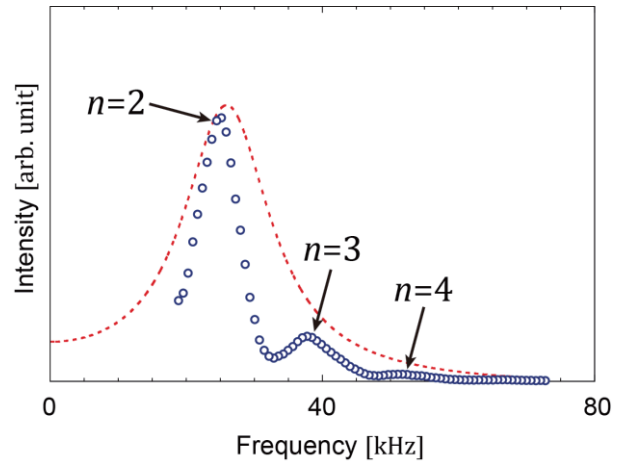


Fig. 2 Ripplon power spectrum obtained for distilled water surface in the cavity at the scattered angle of 0.28° . The red curve, observed on the free surface of same liquid at the same scattered angle, gives the envelope of the resonant peaks.

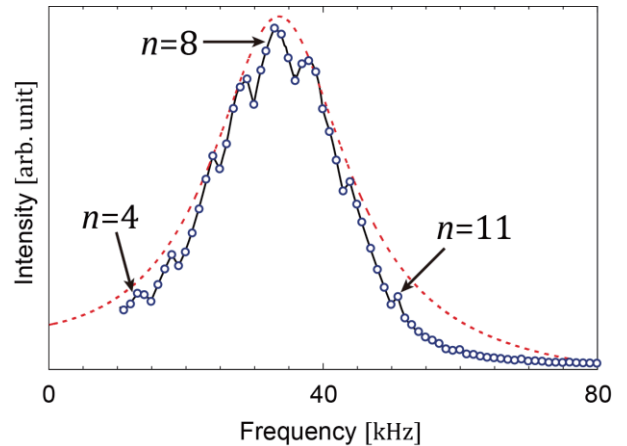


Fig. 3 Ripplon power spectrum obtained for distilled water surface in the cavity at the scattered angle of 0.78° . The red curve, observed on the free surface of same liquid at the same scattered angle, gives the envelope of the resonant peaks.

instrumental band of the detection. We find that the resonant frequencies of the spectrum well agree with theoretical calculation.

References

1. K. Sakai, P. K. Choi, H. Tanaka, and K. Takagi: Rev. Sci. Instrum., **62** (1991)1192.
2. J. R. Sandercook: Phys. Rev. Lett., **29** (1972) 1735.
3. K. Sakai, K. Hattori, and K. Takagi: Phys. Rev. B, **52** (1995) 9402