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Behavior of Elastic and Dielectric Modes in Electromechanical Coupling System from the Viewpoint of Comparing to Conventional Quantum Mechanics

従来の量子力学との比較の観点からの電気機械結合系における弾性及び誘電モードの振舞

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1. Introduction

The quantum mechanics is a fundamental physical theory and has been utilized widely for understanding various physical phenomena.

According to J. J. Sakurai's textbook, the quantum theory includes the classical theory within it, describing as the following: "It (quantum physics) starts with a framework so unlike the differential equations of classical physics, yet it contains classical physics within it. It provides quantitative predictions for many physical situations, and these predictions agree with experiments. In short, quantum mechanics is the ultimate basis, today, by which we understand the physical world." [1] "Classical mechanics can be derived from quantum mechanics, but the opposite is not true." [2] Therefore, it is desirable to utilize the concept of quantum theory widely also in the field of ultrasonic electronics.

In the field of engineering (for example, control engineering), the "classical theory" means the framework of analysis mainly in the frequency domain, based on Newtonian mechanics and the treatment of lumped circuit parameters. For example, in dealing with the characteristics of the electromechanical vibration system in which the elastic phenomenon interacts with the dielectric phenomenon, the classical framework can calculate the relationship between electrical resonance frequencies and antiresonance frequencies, where the existence of lumped-parameter dielectric capacitance components is inevitable.

However, as pointed out in ref. 3, this classical framework has some problems when the analysis is performed in the time domain; that is, when transient or impulse response of the system is calculated. Unfavorable results are caused by the existence of dielectric capacitance and the dissipation (loss). Especially, the difficulty caused by the existence of dissipation suggests the application limitations of classical Newtonian mechanics.

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In order to settle these difficulties, the distributed-parameter-based treatment of vibration analysis has been developed without using any lumped parameter elements,[3-13] in which the "energy mode" has a kind of phase factor, propagates and interferes with each other in a manner of probabilistic superposition. In this method, the dissipation can be treated reasonably. The concept of energy mode is derived from Dirac's "complex dynamical variable" [14], in which its magnitude is proportional to the square root of stored energy. The interaction process between the elastic and dielectric phenomena is treated by considering non-unitary processes (with dissipation) between the elastic energy mode and dielectric energy mode, while the "conventional" quantum theory usually treats *unitary* processes.

In this study, the similarity and difference between the "conventional" quantum theory and author's method are discussed from the viewpoint of analyzing the electromechanical coupling system.

2. Comparison to Conventional Quantum Mechanics

In quantum mechanics, a state vector (ket vector) changes as

$$|a, t\rangle = U(t, t_0)\,|a, t_0\rangle \tag{1}$$

in the Schrödinger picture (not Heisenberg picture), where $|a, t_0\rangle$ is an initial state vector at time t_0, $U(t, t_0)$ is a unitary matrix for the time evolution from time t_0 to t, and $|a, t\rangle$ is the state vector at time t. (By considering the time evolution of U in infinitesimal time, Schrödinger equation can be derived.[2])

Similar formulation is adopted to treat the energy mode in the present study. The energy mode $\boldsymbol{\eta}$ is calculated as

$$\boldsymbol{\eta} = \Sigma_{\text{all paths}} K_{\text{out}} \,(A_0 + RA_0 + R^2A_0 + \ldots)\, K_{\text{in}}\, \boldsymbol{\eta}_0 \tag{2}$$

in an electromechanical coupling vibration system, where η_0 is an initial seed value of energy mode, K_{in} and K_{out} are matrices for input and output processes, respectively, A_0 and R are non-unitary matrices of initial term and common ratio, respectively, of the infinite geometric series, which reflect the boundary condition of the system, and the summation is taken over all possible spatial paths for the energy mode in a probabilistic manner.

The infinite geometric series in eq. (2) corresponds to the summation of energy mode over all time (t = 0 to ∞), which means that time is indefinite. From the uncertain principle between time and frequency, indefinite time gives a definite frequency, and therefore, eq. (2) provides characteristics of the system at a definite frequency. From the poles in eq. (2), the transient or impulse response of the system in the time domain is obtained.[3] Some interaction coefficients between the elastic and dielectric modes are included in A_0 and R. In eq. (2), the dissipation in the system can be dealt with reasonably. If the dissipation did not exist, the infinite series in eq. (2) would diverge to infinity, which does not express the real physical situation.

The formulation with regard to the summation over all spatial possible paths in eq. (2) is similar to Feynman's formulation of path integral. The superposed phase factor in Feynman's formulation has the form of

$$\exp(jS/\hbar), \tag{3}$$

where $\hbar$ is Planck's constant h over 2π (Dirac's constant), S is the time integral of Lagrangian, and j is the imaginary unit.

In the method adopted in this study, the superposed energy modes have the following forms of phase factor:

$$\text{Elastic mode:} \quad \exp(-j\omega T - \alpha_e), \tag{4}$$

$$\text{Dielectric mode:} \ \exp(-j0 - \alpha_d) = \exp(-\alpha_d), \tag{5}$$

where ω is an angular frequency of the energy mode as a wave, T is propagation time for the elastic mode to pass through a spatial domain in the system, and α_e and α_d are attenuation factors of elastic and dielectric modes, respectively, on the spatial domain.

The superposition of elastic mode provides resonance characteristics in the frequency domain, and its spatial distribution corresponds to an eigenvector of resonance mode. On the other hand, the superposition of dielectric mode provides non-resonance characteristics and its spatial distribution has a form that is composed of some spatial delta functions; that is, dielectric energy mode behaves in the way it concentrates in infinitely small areas in the system at any and all frequencies. The interaction process between such elastic and dielectric modes provides physically reasonable results.[4] The forms in eqs. (4) and (5) do not appear in the "conventional" quantum mechanics.

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