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Accuracy of Rayleigh Wave Analysis Using The Moving Particle Simulation Method

MPS 法による Rayleigh 波解析とその精度の検討

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1. Introduction

MPS is a mesh-free numerical technique proposed by Koshizuka et al. [1],[2]. The 'approximation of a continuum by particles' and a 'Lagrangian numerical formulation' make MPS an easy way of treating the nonlinear dynamics of phenomena such as plastic deformations, destructions, and the breaking of waves [3]-[5].

We have applied MPS to wave phenomena in the past; however, the application of this technique to waves is in its infancy [6],[7]. We confirmed the a ccuracy of linear 'scalar' wave analysis using MPS [8]. Elastic wave nonlinear problems have already been analyzed using MPS [9],[10]. However, the b oundary conditions, especially the use of a free bou ndary, have not been discussed.

We focused our attention on Rayleigh waves. The relationships between the phase velocity of Rayleigh waves, the degree of randomness of the particl e distribution, and the radius of the weight function are investigated in detail.

2. Analysis model

Stainless steel (SUS304) was selected as the material to be analyzed. Its properties are shown in **Table I**. The configuration of the model used to calculate the phase velocity of the Rayleigh wave is shown in **Fig. 1**.

As shown in **Fig. 2**, the distance between adjacent particles is L_0, assuming uniformity. L_0 and the discrete time interval Δt are shown in **Table II** alongside the Courant number, where c_p is the phase velocity of the longitudinal waves in the stainless steel.

The wave source, which is a displacement at the point of input, is loaded on the center of the upper surface shown Fig. 1. The shape of the wave source is a sine wave pulse defined below.

$$\exp\left(\left\{\frac{t-b}{b}\right\}^2 \times \ln(10^{-3})\right) \times \sin(2\pi f t) \qquad (1)$$

Here t is time, $b = 6.45 \times 10^{-6}[s]$ and the frequency f = 1.24 MHz.

The phase velocity of the Rayleigh waves in an isotropic solid can be calculated analytically as

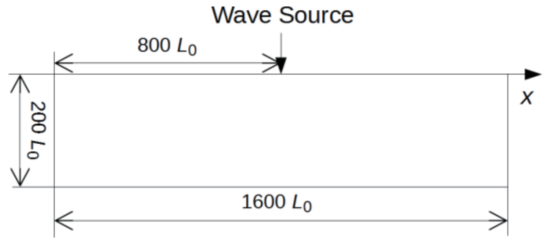

Fig. 1 Stainless steel model analyzed and input point used for verifying the accuracy of two-dimensional calculations carried out with MPS. All of the material's surfaces are free surfaces. It is assumed to be infinite perpendicular to the x–y plane. L_0 = 0.05 mm is the initial distance between adjacent particles.

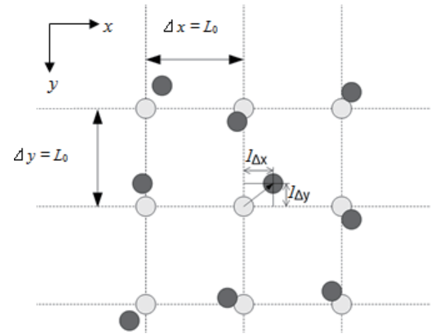

Fig. 2 Distributions of particles used by MPS. The gray particles are aligned regularly. The black circles are particles aligned irregularly. $l_{\Delta x}$ and $l_{\Delta y}$ are the displacements of a particle from its regular position in the x and y directions, respectively.

Table I. Material constants for stainless steel (SUS304).

Material constants	
Density ρ [kg/m^3]	7910
P wave velocity [m/s]	5790
S wave velocity [m/s]	3100
Rayleigh wave velocity [m/s]	2840

Table II. Constant parameters in the MPS calculation.

Constant Parameters	
$L_0 = \Delta x = \Delta y$ [mm]	0.05
Δt[ns]	0.86
$c_P/(L_0/\Delta t)$	0.1

follows [11].

$$\left(\frac{c_R}{c_S}\right)^6 - 8 \times \left(\frac{c_R}{c_S}\right)^4 + (24 - 16k^2) \times \left(\frac{c_R}{c_S}\right)^2 + 16 \times (k^2 - 1) = 0 \quad (2)$$

Where c_r and c_s are the phase velocities of the Rayleigh and shear waves, respectively. $k^2 = (1 - \nu)/2$, where ν is Poisson's ratio. It is well known that Eq. (2) has a real root and c_r is a little smaller than c_s. The calculated value of c_r is shown in Table I.

We investigate the relationship between the phase velocity of the Rayleigh waves and the particle distribution. To do this we introduce the parameter α as follows. The particles represented by black circles are randomly distributed as in Fig. 2. $l_{\Delta x}$ and $l_{\Delta y}$, as shown in the figure, are the displacements in the x and y directions, respectively, from the equilibrium position (indicated by the ash gray circles). $l_{\Delta x}$ and $l_{\Delta y}$ are determined in the range of $-\alpha \times L_0 \leq (l_{\Delta x}\ and\ l_{\Delta y}) \leq \alpha \times L_0$.

3. Computations and discussions

We consider the situation in which the particles are aligned regularly. We investigated the phase velocity of the Rayleigh waves for various radii of the weight function. Here, we introduce the error ratio below.

$$error = \frac{calculated\ velocity - theoretical\ velocity}{theoretical\ velocity} \times 100[\%] \quad (3)$$

In addition, we adopt the parameter β which is defined as $r_e = \beta \times L_0$. In **Fig. 3**, these error ratios are clearly very small, when the parameter α is 0. It is interesting that as β increases, the phase velocity decreases.

Fig. 3 shows the dispersion characteristics versus α for various values of the parameter β. In Fig. 3, as the degree of randomness α increases, the error ratio of the phase velocity of the Rayleigh wave increases negatively (i.e., the wave is delayed).

Furthermore, if the distance of a random particle from a particle in an equilibrium position is sufficiently small, i.e., about $L_0/10$, the waves propagate accurately in terms of both the phase velocity and the waveform at least β is in the range of 2.1 to 5.6.

4. Conclusions

MPS was applied to the numerical analysis of elastic waves. The phase velocity of Rayleigh waves was investigated in detail. It was found that if the distance of a random particle from a particle in an equilibrium position is sufficiently small (about $L_0/10$), waves propagate accurately in terms of both the phase velocity and the waveform at least β is in the range of 2.1 to 5.6.

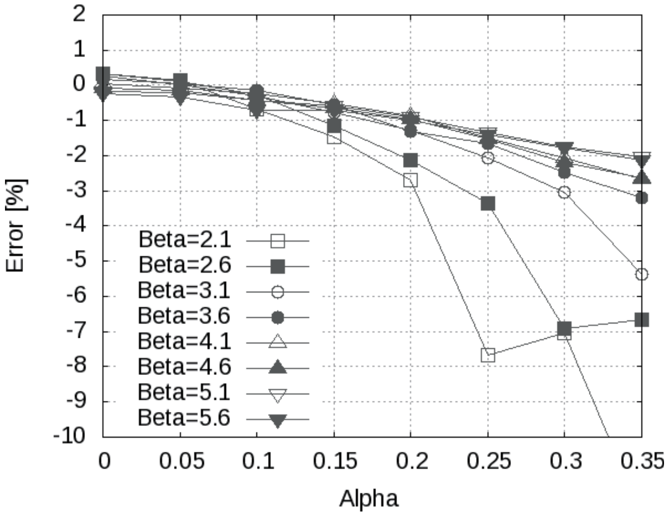

Fig. 3 Error ratio of the phase velocity of the Rayleigh waves versus α. The equation for deriving the error ratio is given by Eq. (3).

In addition, we clarified the influence of the radius of the weight function on the dispersion characteristics of the surface acoustic wave velocity. As the radius increased, the phase velocity delay of the Rayleigh wave changed less with the randomness of the particle distribution.

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