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Modeling of Symmetric Boundary Conditions of Free Vibration Problems for FD-TD Analysis with Staggered Grids with Collocated Grid Points of Velocities
対称境界条件と SGCV を用いた FD-TD 法による固有振動解析

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1. Introduction

The staggered grid with the collocated grid point of velocities (SGCV) was presented for the finite-difference time-domain (FD-TD) method to model propagation of elastic waves in anisotropic solids[1)], and was applied to resonance frequency analysis of a Lamé mode resonator on an isotropic solid to demonstrate the simply imposed boundary conditions on free surfaces[2, 3)]. The results showed that accuracy of the SGCV was comparable to the ones with the conventional staggered grid[4)] with the stress-imaging technique[5, 6)].

For resonant mode analysis, particle velocity and stress fields are either symmetric or antisymmetric with respect to axes of symmetry. Although imposing appropriate symmetric boundary conditions on axes of symmetry can be effective to launch a targeted mode, the symmetry boundary condition has not been implemented in the FD-TD method with the SGCV.

In this paper, symmetric boundary condition is implemented in the FD-TD method with the SGCV, which uses a scheme of second-order accuracy in the time and spatial differences. Bi-linear interpolation with four-adjoining grids is used to evaluate the gradients of particle velocity on grids just inside the free-surface boundaries[2, 3)]. Numerical results show the validity of the symmetric boundary condition. It is also shown that the computational time is reduced due to the reduction of computational domain.

2. FD-TD models with the Symmetric Boundary Condition

We consider a two-dimensional square Lamé mode resonator with a side length of L on an isotropic solid. The resonator is placed on the *x*-*y* plane of a Cartesian coordinate, whose origin is taken on the center of the resonator. The resonator is discretized with square SGCVs with side a length of Δ as shown in **Fig. 1**.

Fig. 2 shows FD-TD models of the resonators discretized with the SGCV. Symmetric boundary conditions are imposed on the symmetry axes and the free surface boundary conditions are imposed on the other edges of the resonator. In Fig. 2 (a), the symmetry axis is $y = 0$ so that the computational domain is reduced to the half of the resonator:$|x| \leq L/2, 0 \leq y \leq L/2$. In Fig. 2 (b), the symmetry axes are $x = 0$ and $y = 0$ so that the computational domain is reduced to the quarter of the resonator:$0 \leq x, y \leq L/2$.The particle velocity components, $v_x(x, y)$ and $v_y(x, y)$, located on the lines of $x = -\Delta/2$ and $y = -\Delta/2$ are added to implement the symmetric boundary condition without a modified FD-TD procedure.

The symmetric boundary conditions for symmetric modes are expressed as $v_x(x, -\Delta/2) = v_x(x, \Delta/2)$ and $v_y(x, -\Delta/2) = -v_y(x, \Delta/2)$ for the boundary on $y = 0$ and $v_x(-\Delta/2, y) = -v_x(\Delta/2, y)$ and $v_y(-\Delta/2, y) = v_y(\Delta/2, y)$ for the boundary on $x = 0$. Note that for the stress components, the standard time-updating procedure[2)] with these external particle velocity components automatically satisfies their symmetry conditions.

3. Numerical Results

We consider the resonator with Poisson's ratio of 0.25 and with the fundamental resonance frequency, f_1, of 1 MHz[2)]. In this paper the Courant number is taken as $R = v_p \Delta_t / \Delta = 0.5$, where v_p, and Δ_t are, respectively, the phase velocity of the P-wave in the solid, and the time interval.

To analyze the resonance frequency, a vibration is given on a $(x_s, y_s) = (3L/8, 3L/8)$, and the time response is observed on $(L/8, L/8)$. The vibration is a sine-modulated Gaussian pulse with the center frequency f_1 given as $v_x(x_s, y_s) = v_x(x_s, y_s) + \sin(2\pi f_1 n \Delta_t) \exp[(n - 150N_l)^2 / (50N_l)^2]$, where n is a time step number and $N_l = L/\Delta$ is a number of cells in the *x*- and *y*-directions. The FD-TD calculations were carried

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out for three models 1) with the symmetry condition on $y=0$ [half domain, Fig. 2 (a)], 2) with the conditions on $x=0$ and $y=0$ [quarter domain, Fig. 2 (b)], and 3) without the condition [whole domain, $|x|,|y| \leq L/2$]. In our calculations, total numbers of time steps were taken as $2^{12}RL/(v_p\Delta_t)$.

After an FD-TD calculation, the discrete Fourier transform is applied to the observed time responses of $2^8 RL/v_p \leq t \leq 2^{12}RL/v_p$ to extract the resonance frequency, f_{DFT}. **Fig. 3** shows the normalized resonance frequencies as a function of N_l. We note that the results for all models are identical. We can see that the normalized resonance frequencies converge to 0.9999 and that the results agree very well with f_1.

Fig. 4 shows the normalized computational time, which is defined as $t_i(N_l)/\,t_w(N_l)$ $(i=w,h,q)$, for each model as a function of N_l. Here, $t_i(N_l)$ $(i=w,h,q)$, respectively, denote the computational times with whole-, half-, and quarter-size of domains. Our codes were run on MATLAB in single-thread mode for more accurate profiling. We can see that computational time is effectively reduced for larger values of N_l due to the reduction of the computational domain with the symmetric boundary conditions. For large models with $\log_2 N_l \geq 8$, the normalized computational times are slightly smaller than 0.5 and 0.25, respectively, for the analyses with half- and quarter-size of computational domains. The authors think that the reduction of computational domains improved cache-hit ratio of the codes.

4. Conclusions

In this paper, the symmetric boundary conditions were implemented in the FD-TD method with the SGCV. It was applied to resonance frequency analysis of a Lamé mode resonator. The results showed the validity of the symmetric boundary condition. It was also shown that the computational time is reduced due to the reduction of computational domain.

References

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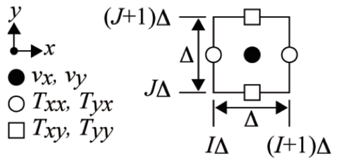

Fig 1 A unit cell of the SGCV.

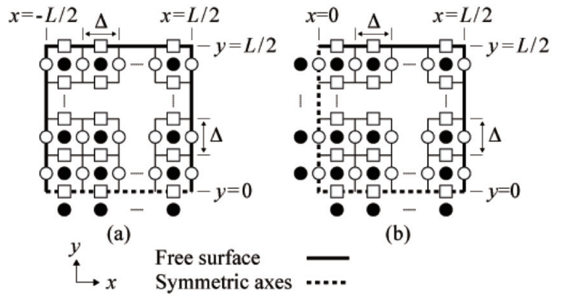

Fig. 2 A Lamé mode resonator discretized with the SGCV. The symmetric boundary conditions are imposed for analysis with (a) half and (b) quarter regions

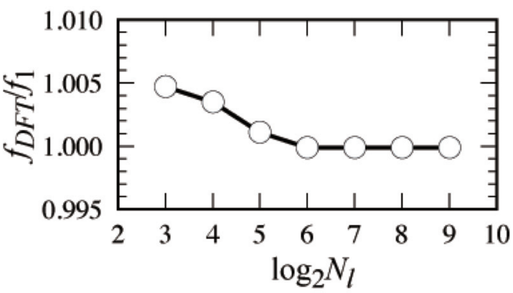

Fig. 3 Extracted normalized resonance frequencies with the whole-, half-, and quarter-size of computational domains. Note that results for all models are identical.

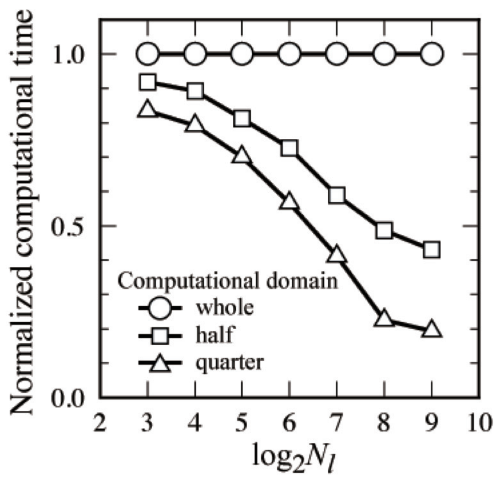

Fig. 4 Normalized computational time for whole-, half-, and quarter-size of computational domain as a function of N_l. Note that the total number of unknowns is in $O(N_l^2)$.