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Piezoelectric Oscillation Circuit Analyses in Consideration of Non-linear Properties
非線型性を考慮した圧電発振回路解析

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1. Introduction

In the piezoelectric oscillator including the crystal oscillator, the phenomenon that oscillatory frequency fluctuates by line voltage variation is known widely. The analytical method for this phenomenon was suggested, but expression of the real frequency change was not enough until now either [1]. The reasons include two problems such as the limit of the point where a non-linear property of the piezoelectric resonator was not considered [2]and the calculation dynamic range. With the above-mentioned non-linear property, a resonance frequency of the piezoelectric resonator showed a phenomenon to vary with drive power, but enough examination was not formed about the circuit model. In addition, working regiones of the crystal oscillators are MHz - GHz regiones, and the amount of frequency change validationes are several number of ppm. Furthermore, a large dynamic range was required for calculation unless we performed close standardization because magnitude of the series-capacitance to use with the equivalent circuit of the vibrator was a "femto farad".

This paper focused its attention on power - non-linear properties of the piezoelectric resonator and designed the numerical model that could express the change of the resonance frequency for the drive power. Furthermore, it examined the validity mentioned above and it considered power - non-linear characteristic on the activity circuit side and experimented on the simulation about the change of the oscillatory frequency at the time of the operation.

As an example, it performed the oscillatoin frequency change simulation for the power supply voltage change of the colpitts crystal oscillator using the AT-cut quartz resonator and made clear the advantage of the proposal method from the result.

2. Non-linear Characteristics of Piezoelectric Resonators

It is studied the non-linear properties of the vibration body for a long time, and the research on power-non-linear property (current square properties) of the crystal resonators have the old history [3].

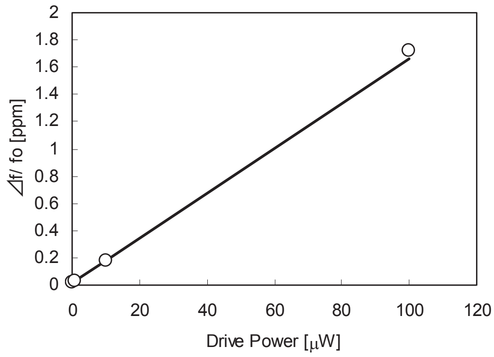

Fig. 1. Example of frequency-power relation of AT-cut quartz crystal resonator.

Figure 1 shows power properties of the representative AT-cut crystal resonator. It understands that resonance frequencies increase in proportion to the power that square of the driving current is added to resonator from the figure. It can explain the phenomenon of Fig. 1 from a rigid nonlinear forced oscillation theory [4]. The vibration displacement quantity of the resonator shall be proportional to added voltage or current according to a piezoelectric basic expression [5]. The large case is the next expression, and vibration displacement is expressed by the non-linear Duffing equation [4] including the shown term of a high order.

$$\frac{d^2u}{dt^2} + \omega^2 u + \beta u^3 = 0 \tag{1}$$

The general solution of this formula is given by

$$u = A\cos\gamma t + \frac{1}{4}\beta\frac{1}{9\gamma^2 - \omega^2}A^3\cos 3\gamma t \tag{2}$$

Here $\gamma^2 = \omega^2 + \frac{3}{4}\beta A^2$ and A express the vibration amplitude.

While it can consider it to be constant because ω is bigger enough than $\Delta\omega$ as far as a frequency change is small, then

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$$\Delta\omega = \frac{3}{8}\frac{\beta}{\omega}A^2. \tag{3}$$

It understands that the resonance frequency amount of change of the vibration body is proportional to square of the amplitude to be clearer than this formula.

3. Oscillation Circuit and High Frequency Model

Figure 2 is a colpitts crystal oscillation circuit using NPN and BJT. **Figure 3** shows a high frequency model of Fig. 3.

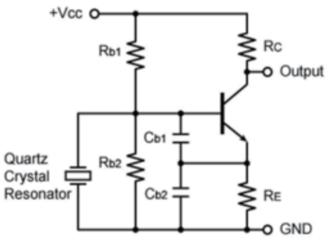

Fig. 2. Colpitts quartz crystal oscillation circuit.

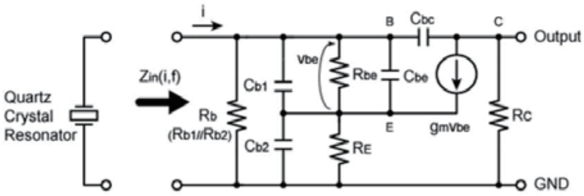

Fig. 3. High-frequency model of Colpitts circuit.

$$Z_1 = \frac{1}{\frac{1}{R_{be}} + j\omega\left(C_{b1} + C_{be}\right)} \qquad Z_2 = \frac{1}{\frac{1}{R_b} + j\omega C_{b2}}$$

$$Z_3 = R_c + \frac{1}{j\omega C_{bc}} \tag{4}$$

It can express impedance $Z_{in}(i,f)$ which it anticipated from a crystal resonator connection terminal by the next formula using Z_1 - Z_3.

$$Z_{in}(i,f)$$
$$= R_b // Z_3 // \left(Z_1 + Z_2 + g_m \cdot Z_1 \cdot Z_2\right) // \frac{\left(Z_1 + Z_2 + g_m \cdot Z_1 \cdot Z_2\right) \cdot Z_3}{g_m \cdot Z_1 \cdot R_c} \tag{5}$$

A g_m in formula (5) uses nonlinear coefficient k_x expressing nominal value g_{mo} of g_m and the bias change by the high-frequency current component and it can be expressed by,

$$k_x = \frac{2}{i_1 Z_1} \cdot \frac{kT}{q} \cdot \frac{I_1\left(i_1 \cdot Z_1 \cdot q/kT\right)}{I_0\left(i_1 \cdot Z_1 \cdot q/kT\right)} \tag{6}$$

An i_1 is current to flow in Z_1, I_0 and I_1 are zeroth and primary first order the modified Bessel function, k is Boltzmann constant, T is absolute temperature, q is unit charge. Resonator non-linear model and $Z_{in}(i,f)$ were connected to serial to find a state to be satisfied in oscillation condition. When a real part and the imaginary part of the impedance were driven in high frequency current source, it searched for a point of the absolute impedance equal zero.

4. Simulation results

Table 1 shows constant of the Colpitts circuit used for numerical experiment. Equivalent constant of the crystal resonator (20MHz) is shown in **table 2**. **Figure 4** is a simulation result of the oscillator frequency change for the supply voltage. It was shown that a large oscillatory frequency change was provided by using a crystal resonator model in consideration of power - non-linear properties.

Table 1. Circuit parameters of Colpitts oscillator.

R_{b1} (kΩ)	R_{b2} (kΩ)	R_c (kΩ)	R_e (kΩ)	C_{b1} (pF)	C_{b2} (pF)
10.0	10.0	0.56	1.0	100	100

Table 2. Equivalent parameters of quartz unit.

C_0(pF)	R_1(Ω)	L_1(mH)	C_1(fF)
1.95	10.6	8.98	7.06

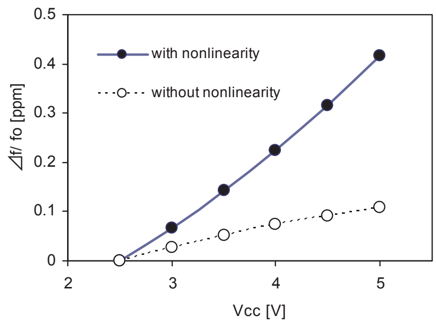

Fig.4. Calculated characteristics of oscillation frequency vs. power supply voltage.

5. Conclusion

The oscillation circuit analysis in consideration of non-linear properties of the piezoelectric resonator was proposed. As an example, oscillator frequency - supply voltage properties of the crystal oscillator were shown.

Acknowledgment

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