

# Energy Trapping Analysis of Beveled Quartz Resonators with Finite Element Method

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## 1. Introduction

AT-cut quartz resonators utilizing the thickness shear mode (TSM) are used widely as frequency reference in an electronic system [1, 2]. The resonance frequency of TSM can be estimated with the ratio of wave velocity to two times the plate thickness. The sandwich structure of electrode-quartz-electrode leads to concentration of the thickness shear mode inside electrode area. However, the boundary condition of a quartz plate still affects tremendously the TSM characteristics [3]. For example, the unwanted flexure and loss would be induced by the supports, which attach the quartz plate to the substrate.

To improve the attachment effects, a quartz plate with beveled edge was proposed. The literatures [4-6] show the steeper the bevel slope, the better the confinement of the TSM energy. However, more beveling makes the excitation weaker and cost larger. In this paper, the mode shape and the resistance of AT-cut quartz resonators are analyzed with and without the beveled edges using the finite element method. As shown in **Fig. 1**, the lossy epoxy attachments are considered in the simulation. The finite element software, COMSOL Multiphysics, is used to analyze the quartz plates under different beveling time. The mode shapes are visualized to show the bevel effect on the generated flexure component. The frequency responses and resistances of the beveled quartz resonators are also calculated and compared to the experiment results.

## 2. Unbeveled quartz resonator

The unbeveled quartz resonator was characterized by calculating its eigenfrequency and frequency response. The geometry is shown in **Fig. 2**. The material constants of quartz crystal used in the simulation are from Bachmann's paper in 1958 [7]. The material constants of the epoxy attachments are listed in **Table I**. The electrode is Au film whose constants can be found in COMSOL material library. Besides, the loss constants of quartz crystal, Au film, and epoxy are  $10^{-6}$ ,  $10^{-5}$ , and  $10^{-4}$ , respectively.

According to simulation results, the resonance

frequency of the TSM mode is around 16.41 MHz when the quartz plate is 0.1 mm in thickness. From **Fig. 3(a)** and **3(b)**, the flexure component in TSM appears stronger when the epoxy attachments are introduced, which indicates the boundary of the quartz plate seriously affects the resonance characteristics. **Figure 4** is the frequency responses of the resonator from simulation and experiment. Both results show two peaks around the resonance frequency.

## 3. Beveled quartz resonator

4 beveled quartz resonators were taken into the simulation. The geometry of beveled edge is illustrated in **Fig. 1**. Practical bevel sizes were measured by optical technique, and the results are sorted in **Table II**. The bevel arc is elliptic in the simulation. **Figure 3(c)** shows the amplitude of the total displacement of the 15-hour beveled quartz resonator with epoxy attachments. The 15-hour beveling suppresses significantly the flexure and confines well the TSM energy under the electrode.

Thanks to the beveled edge, the mode energy is confined well inside the central area of the quartz plate, and thus the energy leakage from the attachment is reduced significantly. **Table III** lists the energy loss characteristics of the quartz resonators under different beveling time, like imaginary part of eigenfrequency and resistance at resonance frequency. One can find the longer the beveling time, the smaller the resistance. The trend is in good agreement with the experiments in Ref. [8].

## 4. Conclusions

In this paper, the mode shape, frequency response, and resistance of AT-cut quartz resonators with and without the beveled edges were analyzed using the finite element method. Especially, the lossy epoxy attachments were considered in the model. Results show that due to the beveled edge, the mode energy would be confined well inside the central area of the quartz plate, and thus the energy leakage from the attachment would be reduced significantly. Besides, the resistance at resonance frequency decreases with the increasing beveling time. The trend is consistent with the measured results, confirming that the proposed

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model is useful to design practical quartz resonators.

### Acknowledgment

The authors gratefully acknowledge the financial support from National Science Council of Taiwan under the grant MOST 103-2221-E-036-010 and Tatung University in Taiwan through the grant B103-E05-042.

### References

1. D. E. Nivens, J. Q. Chamber, T. R. Anderson, and D. C. White: Anal. Chem. 65 (1993) 85.
2. R. Aigner: IEEE IUS Proc. (Beijing, CHN, 2006) 582.
3. M. Tanaka, T. Imai, and Y. K. Yong: IEEE IFCS (Florida, US, 2003) 709.
4. P. Li, F. Jin, and J. Yang: IEEE IUFFC 59 (2012) 1006.
5. W. Wang, R. Wu, J. Wang, J. Du, and J. Yang: IEEE IUFFC 60 (2013) 1192.
6. L. Li, M. Esashi, and T. Abe: Appl. Phys. Lett. 85 (2004) 2652.
7. R. Bechmann: Phys. Rev. 110 (1958) 1060.
8. S.-Y. Pao, W.-Y. Chang, B. Hsu, Y.-T. Lai, Y.-Y. Chen, T. Lin, and M.-K. Chao: IEEE IFCS (Taipei, TWN, 2014) 358.

Table I Material constants of epoxy in simulation.

Young's modulus (GPa)	3.9379
Poisson ratio	0.3769
Density (kg/m <sup>3</sup> )	1157.8
Loss	10 <sup>-4</sup>

Table II The edge geometry of the quartz plates used in simulation (in mm).

	0 hr.	5 hrs.	10 hrs.	12 hrs.	15 hrs.
<i>a</i>	0	0.00265	0.0136	0.0129	0.01265
<i>b</i>	0	0.183680	0.249277	0.297016	0.454792

Table III Loss characteristics of quartz resonators under different beveling time.

	Imaginary part of eigenfrequency (Hz)	Resistance <i>R</i> (ohm) (simulation)	Resistance <i>R</i> (ohm) (experiment) Ref. [8]
0 hr.	249.43	84.03	119.1
5 hrs.	67.47	14.83	40.2
10 hrs.	52.50	10.65	22.7
12 hrs.	34.02	7.29	21.6
15 hrs.	10.95	2.42	17.3

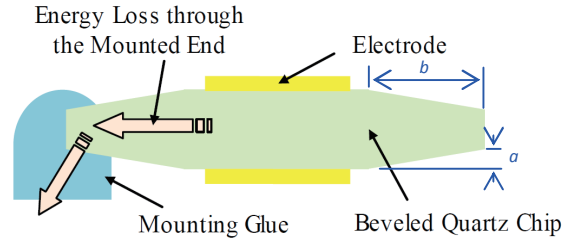


Fig. 1 Illustration of beveled quartz resonator.

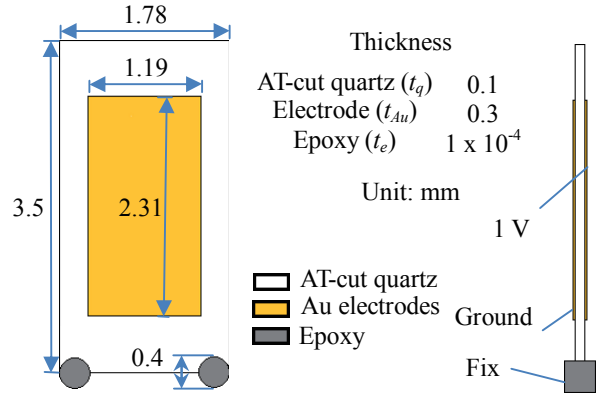


Fig. 2 Geometry of quartz crystal used in simulation.

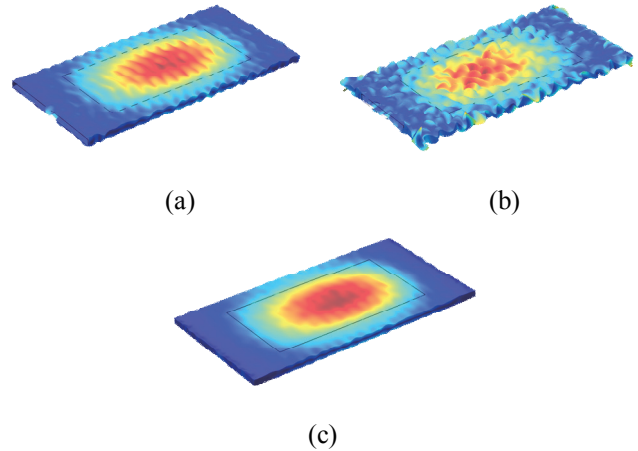


Fig. 3 TSM mode shapes of (a) unbeveled quartz plate without epoxy, (b) unbeveled quartz plate with epoxy, and (c) 15 hours beveled quartz plate with epoxy.

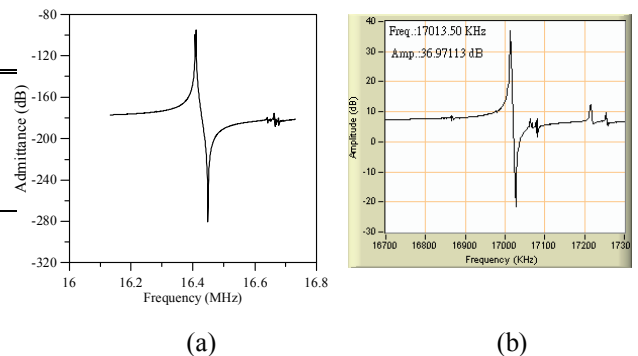


Fig. 4 Frequency responses of unbeveled quartz plate with epoxy by (a) simulation; (b) measurement.