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Nonlinear coefficients in lead-free materials

圧電振動における非線形性に関する研究

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1. Introduction

Hard-type piezoelectric materials need to be driven under a high voltage; however, the performance of piezoelectrics is limited by nonlinear phenomenon [1]. Frequency response curves of these properties deform under high voltage. Jumping phenomenon appears beyond the threshold electric field strength, accompanied by a hysteresis of properties when they are measured in different sweep directions. Systematic research on nonlinearity in resonators was carried out, in which nonlinear terms were proposed as a function of current and nonlinear coefficients were obtained [2]-[3]. In high power applications, continuous voltage is usually applied on the devices, which results in the heat generation. Transient measurement method [4] has been used to separate the temperature effect from conventional measurement results, especially under high driven voltage. The essence of these methods is short measuring time and negligible heat generation.

In this study, prior to the investigation of the nonlinearity in lead-free CuO-$(K_{0.48}Na_{0.52})NbO_3$ transducers, PZT transducers were investigated. At first the burst mode measurement was utilized to measure the vibration velocity of PZT transducers after excited by burst voltage. The results suggest the decaying rate and resonant frequency to be functions of velocity amplitude, from which nonlinear model can be derived. This model enables the curve fitting of admittance curves so that the force factor and nonlinear coefficients were determined.

2. Experimental Procedures

PZT transducers (Fuji Ceramics, C-203 and C-6, 43 mm×7 mm×2 mm) as well as lead-free CuO-KNN transducers (7 mm×2.7 mm×1 mm) poled along the thickness direction were utilized in our experiments, which were excited in longitudinal mode with 31 effect. CuO-KNN disks were fabricated via a hydrothermal method [5] and then cut into plates. The admittance curves were measured via a frequency response analyzer (NF FRA5097). The transducers were driven by the output signal of NF FRA5097 after amplified by a power amplifier (NF HSA4052). In the burst mode measurement the samples were driven by a function generator (NF WF1974) connected by the same power amplifier (NF HSA4052). The transducer vibration velocity was measured by a Laser Doppler Vibrometer (Polytec NLV-2500). The driving voltage and the vibration velocity were monitored and recorded by an oscilloscope (Agilent DSO5034A). The driven signal was set in burst trigger mode. After each trigger signal, subsequent to sinusoidal waves (1000 cycles), the electrical driving port was shortened. The curve fitting in this study was carried out in the software Igor Pro (Version 6.3.3.1).

3. Results and discussions

In the burst mode measurement, after excited by the burst signal (1000 cycles), the transducers began free vibration (not shown here). The velocity decreased after releasing the burst voltage and in soft type transducer it decreases much faster than that of hard transducer. High voltage results in larger decaying rate at the beginning than low voltage does. Considering the phase delay, the curves can be fitted using the following function:

$$v(t) = v_0 \sin(\omega_r t + \delta)\exp(-\beta(t-\tau)) \qquad (1)$$

where ω_r, β, v, v_0, δ and τ are resonant frequency, decaying rate, vibration velocity, velocity amplitude, phase delay and beginning of free vibration, respectively. Relationships between ω_r, β and v were illustrated in **Fig. 1**. In Fig. 1 (a), the β curves from hard PZT under 10 and 100 $V_{p\text{-}p}$ overlaps with each other, indicating good reliability and repeatability of our measurement. The increasing tendency of β value with velocity suggests a quadratic function, also confirmed by the curve fitting with a zero first order coefficient. Similarly, ω_r is also a quadratic function of vibration velocity (Fig. 1 (b)). Similar relationships work for soft PZT transducer (not shown). Equivalent capacitance C is given by ω_r and equivalent inductance L; equivalent resistance R is determined by L and β:

$$\frac{1}{C} = L\omega_r^2, R = 2\beta L \qquad (2)$$

Conventional LCR equivalent circuit is feasible to describe the circuit property without considering nonlinearity, expressed by:

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt = V \qquad (3)$$

where, $V=V_0\cos\omega t$ and $i=i_0\cos(\omega t+\theta)$ are driving voltage and motional current; θ is the phase delay between current and voltage. The result in Fig. 1 suggests the existence of nonlinearity, we have

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$$L\frac{di}{dt}+R_0 i+\eta i^3+\frac{1}{C_0}\int idt+\xi\omega^3\left(\int idt\right)^3=V \quad (4)$$

Using the expressions of V and i and ignoring the high harmonics derived from the nonlinear terms of equation (4), at last we have

$$\left(-\omega L i_0+\frac{i_0}{\omega C_0}+\frac{3\xi i_0^3}{4}\right)^2+\left(R_0 i_0+\frac{3}{4}\eta i_0^3\right)^2=V_0^2 \quad (5)$$

$$\theta=\tan^{-1}\left[\left(-\omega L+\frac{1}{\omega C_0}+\frac{3\xi i_0^2}{4}\right)\Big/\left(R_0+\frac{3}{4}\eta i_0^2\right)\right] \quad (6)$$

The value of motional current can be determined from (5), which is a third order equation of square of current. Third order equations have three solutions; one or three of them are real solutions. The jumping phenomenon appears when for certain frequency range the equation has three real solutions. Depending on the initial condition, one solution was measured, resulting in the hysteresis between different sweep directions. Motional admittance and total admittance are expressed by

$$Y_m=G_m+jB_m=\frac{i_0}{V_0}\cos\theta+j\frac{i_0}{V_0}\sin\theta \quad (7)$$

$$Y_{total}=Y_m+j\omega C_d \quad (8)$$

The admittance curves of the both transducers were measured by a frequency response analyzer, as shown in **Fig. 2** (points). In Fig. 2 (a), the admittance values around the resonant frequency are not consecutive like other points, usually called jumping phenomenon. In addition, upward sweep and downward sweep measurements gave different results, forming a hysteresis. The admittance curves of soft PZT transducer was shown in Fig. 2 (b); although the resonant peak also shifts to low frequency range, it is different from Fig. 1 (a). Neither jumping phenomenon nor the hysteresis appears.

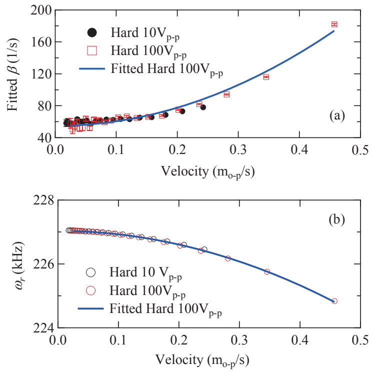

Fig. 1. Fitted (a) β and (b) ω_r of hard PZT as functions of velocity.

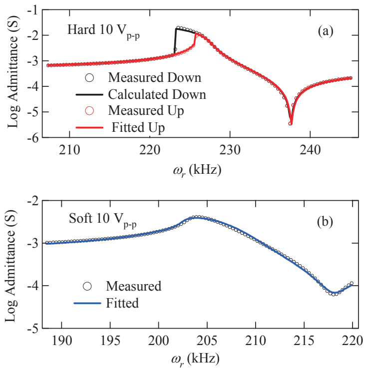

Fig. 2. Measured and fitted admittance curves of (a) hard and (b) soft PZT transducers under 10 $V_{p\text{-}p}$.

Using the model derived above the curve fitting of admittance curves was carried out. For hard transducer, since the admittance hysteresis corresponds to nonlinearity, for the curves measured both in upward and downward sweep the same series of parameters should be used. So the fitting was carried out for upward sweep result and the downward sweep admittance curve was calculated from obtained parameters. As shown in Fig. 2, both the jumping and admittance hysteresis are described very well. The fitted parameters including A, ξ, η, and C_d are shown in **Table I**. Two nonlinear coefficients ξ and η of soft PZT transducer are much larger than the ones in hard PZT transducer, probably due to the lower velocity in soft transducer under the same voltage.

Table I. Force factor and nonlinear coefficients of PZT transducers.

	A (N/V)	β_0 (s^{-1})	η (ΩA^{-2})	ω_0 (kHz)	ξ (ΩA^{-2})	C_d (pF)
Hard	0.153	54	5.07×10^3	227	-9.57×10^4	2.06
Soft	0.191	1037	4.09×10^5	206	-8.83×10^5	3.12

Similarly, abovementioned nonlinear phenomenon appears in CuO-KNN plate transducers, which can be illustrated using the proposed model.

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