

Impact of odd-even phase modulated harmonic components in nonlinearity of shear elasticity

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1. Introduction

A striking effort has been experienced in the field of elasticity imaging over the past decades essentially driven by the need for tissue mechanical characterization for a sophisticated medical diagnosis. Tissue mechanical characterization using shear wave velocity is being researched with great interest for the improvement of the existing underlying pathologies. For this reason, we have introduced a sophisticated method that uses continuous shear wave excitation (CSWE) for estimating the velocity of shear wave propagation inside the tissue using low frequency of external vibration excitation [1,2]. We had proposed a visualization method for the harmonic component of a shear wave that uses the local velocity of the shear wave propagation for reconstructing the phase modulation component and the texture pattern obtained in the local velocity map indicates the existence of nonlinearity of shear elasticity [3].

We had also already proposed a simple vibration-frequency independent hysteretic model which characterizes the nonlinearity of shear elasticity based on the small harmonic phase modulation of displacement [4].

In this paper, we attempt to evaluate the nonlinearity of shear elasticity more precisely by categorizing the phase modulation harmonic components of the displacement into odd and even components. These two components are evaluated individually in order to illustrate their impact for the proposed hysteretic model of shear elasticity.

2. Odd-Even Harmonic Phase Modulation Component of Displacement

In this section we discuss the recovery of the individual odd and even phase modulation of the harmonic component of a shear wave displacement. First the displacement $\varepsilon(x, z)$ due to shear wave propagation at the point (x, z) is estimated by the radio frequency (RF) correlation method. Then the complex displacement is derived by applying the Fourier analysis is as follows:

$$\xi(x, z) = |\xi(x, z)| \exp(j\theta(x, z)) = \mathcal{F}\{\varepsilon(x, z)\}, \quad (1)$$

where \mathcal{F} is the Fourier transform and $\theta(x, z)$ is the

phase of the displacement.

If we assume that there is also an amplitude modulation of the displacement on the shear wave propagation then the amplitude modulated signal can be written as

$$\xi_{(x,z)} = [\xi_0 + \sum_0^n \xi_n \sin(\omega_0 t + \theta_n)] \sin(\omega_0 t + \phi), \quad (2)$$

where n represents the order of harmonic component and ξ_n is amplitude modulation index for the n^{th} order harmonics.

For a very small phase displacement, the amplitude modulation is nearly equals to phase modulation. Therefore the phase modulated small displacement can be split up into odd and even harmonic components. The comb filter is used for splitting the conventional harmonic displacement into odd and even components. Equations (3) and (4) illustrate the equations for odd and even harmonic components of the conventional displacement which is derived from Eq. (2). By using these equations, shear wave local velocity is estimated.

$$\xi_{odd} = \xi_0 + \sum_{n=1}^{\infty} \xi_{2n} \sin(2n\omega_0 t + \theta_{2n}) * \sin(\omega_0 t + \phi) \quad (3)$$

$$\xi_{even} = \sum_{n=0}^{\infty} \xi_{2n+1} \sin((2n+1)\omega_0 t + \theta_{2n+1}) * \sin(\omega_0 t + \phi) \quad (4)$$

Simulations are performed by using Eq. (3) and (4). Figure 1(b) and (d) shows the distorted displacement plot (dotted line) due to the nonlinearity of the force-displacement (F-D) curve shown in Fig. 1(a) and (c) for odd and even harmonics respectively. The symmetrical nonlinearity observed in F-D curve due to odd harmonic component shows similar characteristics of the change in stiffness of the medium under either the application of push or pull force. Whereas the asymmetrical nonlinearity observed in F-D curve due to even harmonic component shows that the characteristics of change in stiffness of the medium under push force changes from that under application of pull force or vice versa. Information from these two completely different characteristics of nonlinearity of shear elasticity for odd and even

harmonic components is expected to establish a sophisticated tool for mechanical characterization of complex tissue structure.

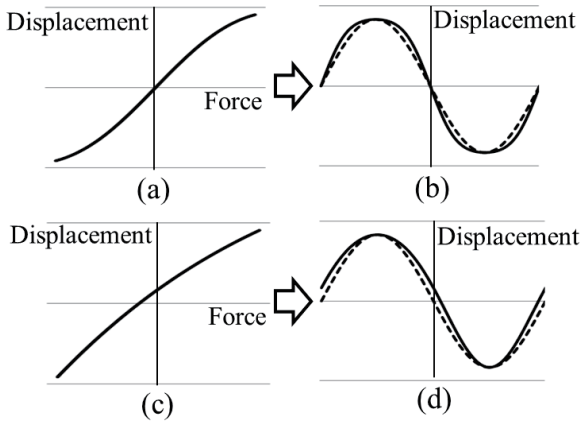


Fig. 1. (a) Symmetrical nonlinearity and (b) corresponding displacement plot (dotted line) for odd harmonic components. (c) Asymmetrical nonlinearity and (d) corresponding displacement plot (dotted line) for even harmonic components.

3. Experimental Results

The Carrageenan gel is used to evaluate the texture pattern of the local velocity map reconstructed from the phase modulated odd and even harmonic components. Figure 2(a), (b), (c) shows the texture pattern of a normalized local velocity map of conventional, odd and even harmonic components respectively obtained for carrageenan gel of 1.3wt% at the vibration frequency of 300Hz. The velocity map is taken within the ROI of $9 \times 8 \text{ mm}^2$. The odd harmonics component consists of the major portion of fundamental frequency component and is very similar to conventional one than the even harmonic components. Therefore the texture patterns in case of local velocity map obtained from the even harmonic components are shrunk than that of conventional and odd harmonic mapping.

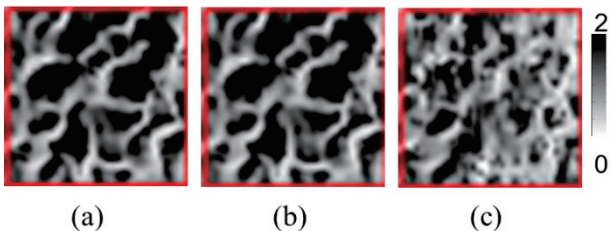


Fig. 2. Normalized local velocity map of carrageenan gel (1.3 wt%) at vibration frequency of 300Hz. (a) Conventional (b) Odd and (c) Even harmonic local velocity map.

Moreover, carrageenan gels of different weight percentage are experimented and their local velocity maps within a ROI are numerically evaluated by

using a histogram map. Figure 3 (a) shows the histogram plot of the conventional, odd and even normalized local velocity map for 1.3 wt% of carrageenan gel. The results shows that the histogram of brightness of normalized local velocity map is almost same for conventional and odd component velocity mapping, whereas it is bit higher for even harmonic component velocity mapping. Also the skewness of the texture pattern evaluated within the ROI of conventional, odd and the even harmonic velocity map of 1.3, 0.9 and 0.5 wt% of carrageenan gel, shown in Fig. 3(b) shows the linear increment of the skewness with the decrement of the wt% of carrageenan gels.

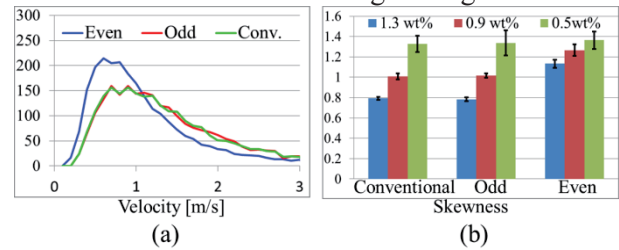


Fig. 3. (a) Histogram of normalized local velocity map for conventional, odd and even harmonic component for 1.3 wt% of carrageenan gel. (b) Skewness of texture patterns of 1.3, 0.9 and 0.5 wt% carrageenan gels for conventional, odd and even harmonic component velocity mapping.

4. Conclusions

In this paper we have discussed about reconstruction of the odd and even harmonic displacement components of the shear wave and its individual velocity mapping.

The local velocity obtained for the odd and even harmonics components are evaluated and compared with the theoretical explanation. The similarity of conventional and odd velocity mapping and the appearances of texture pattern in even velocity mapping, though is the preliminary result, but it is expected to have powerful information about the symmetrical and asymmetrical nonlinearity of the shear elasticity. Moreover the increment of skewness with the decrement of wt% of carrageenan gels for both the even and odd harmonic components acts also an important clue for exploring impact of odd-even harmonic components in shear elasticity.

References

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