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Evaluation of the Skewed Rough Surface Profile based on Johnson Distribution by Ultrasonic Wave Scattering

Muhammad Nur Farhan Saniman[1‡], and Ikuo Ihara[2] ([1]Grad. School of Eng., Nagaoka Univ. of Tech.; [2]Mech. Eng. Dept., Nagaoka Univ. of Tech.)

1. Introduction

The evaluation of skewness-related rough surface of materials is of interest and important in manufacturing process because it affects the contact mechanic of the finish products especially the ones that related to the friction, lubrication, wear and adhesion.[1)] Even though the conventional method such as stylus profilometry has widely been used to characterize rough surfaces, alternative methods based on wave scattering phenomena such as ultrasonic technique have been developed in order to overcome the difficulties of applying stylus method for in-situ or on-line measurements.[2-4)] Although in such ultrasonic method, it is common to assume that rough surfaces have Gaussian distributed height profile, it is not always acceptable for some cases because the non-Gaussian distributed rough surface profiles also often exist when the material surfaces are polished mechanically or treated chemically. Therefore, in this work, Johnson distribution is used to characterize the skewness of the rough surface profiles for a wide range of roughness. Then, the effects of skewness on the reflected amplitude of ultrasonic waves are numerically estimated.

2. Theory

The measure of asymmetry of the height distribution of the surface profile is called skewness *Rsk* and is given as[5)]

$$Rsk = \frac{1}{NRq^3}\sum_{i=1}^{N} h_i^{\,3}\ , \tag{1}$$

where h and N are the deviation of surface point from the mean value of height and the number of data points, respectively. Zero skewness means a symmetry height profile with as many peaks as valleys. Positive skewness means that there are a lot of high peaks while negative skewness means that there are many valleys in the height profile.

In the ultrasonic roughness measurement, Kirchhoff theory has been used to study the scattering phenomena from random rough surface having symmetry height profile. When an ultrasonic wave is reflected from a rough surface at normal specular angle (incident angle θ_i = reflected angle θ_r = 0º), the intensity of the coherent wave component I_{coh} is given by[6)]

$$I_{coh} = I_0 \exp(-2k^2 Rq^2)\,, \tag{2}$$

where I_0 is the reflected wave intensity from a perfect smooth surface, k is the wavenumber and Rq is the root-mean-square roughness.

In order to evaluate an asymmetry rough surface profile, Johnson distribution with four controllable parameters that allows a great flexibility in fitting any kind of skewed rough surface profile is introduced. The probability density function (*pdf*) $p(x)$ and the cumulative distribution function (*cdf*) $P(x)$ of the unbounded Johnson distribution are respectively given by[7)]

$$p(x) = \frac{\delta}{\sqrt{2\pi}}\frac{1}{\sqrt{x^2+1}}\exp\left[-\frac{1}{2}\left\{\gamma + \delta\ln\left(x+\sqrt{x^2+1}\right)\right\}^2\right], \tag{3}$$

$$P(x) = \sum p(x_i)\,, \tag{4}$$

where

$$x = (h-\xi)/\lambda\,. \tag{5}$$

Here, γ, δ, ξ and λ can be calculated from known values of average height h_{mean}, Rq, Rsk and kurtosis Rku based on Tuenter algorithm.[8)] In this study, Rq and Rsk are the controlled parameters while h_{mean} and Rku are fixed to 0 and 3, respectively, which are the values for standard normal distribution. When Rku = 3, $|Rsk|$ is limited to 1.27.

3. Numerical estimation of Rq_{base}

For a skewed rough surface profile, the peaks or valleys of the profile are represented by the tail of the *pdf* and a reflection surface with relatively smaller Rq value (Rq_{base}) than the overall Rq value of the surface profile ($Rq_{profile}$) is dominant in the ultrasonic measurements as shown in **Fig. 1**. By determining the value of Rq_{base}, the effect of skewness to the ultrasonic roughness measurement can be estimated quantitatively. Rq_{base} of the specific $Rq_{profile}$ can be determined by excluding $t \times 100\%$ of the *cdf* that is assumed to be the peaks or valleys of the profile and is calculated from the *pdf* by the following equation:

$$Rq_{base} = \sqrt{\frac{1}{N}\sum_{i=z}\{h_i \cdot p(x_i)\}^2}\ . \tag{6}$$

Here, z is the value of h when $cdf = t$. t is an arbitrary threshold value that can be estimated from

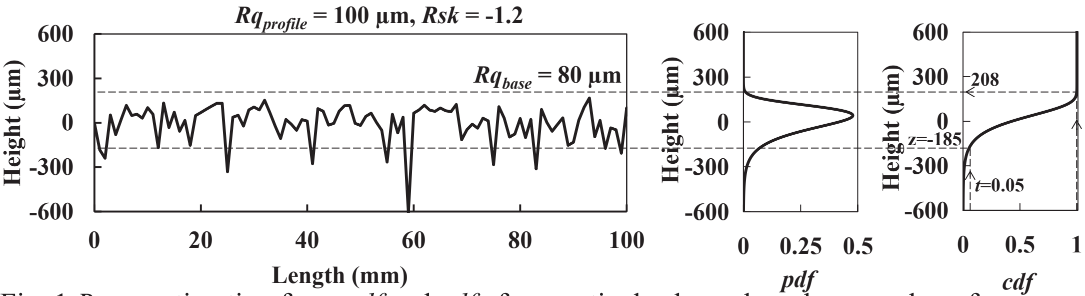

Fig. 1 Rq_{base} estimation from *pdf* and *cdf* of a negatively skewed random rough surface.

the rough surface profile and $t = 0$ means that all the surface points are included in the estimation.

4. Results and Discussion

Based on the above method, the values of Rq_{base} for each $Rq_{profile}$ ranging from 1 to 1000 µm are calculated and then normalized with the corresponding $Rq_{profile}$ value. **Figure 2** shows the changes in the normalized Rq_{base} as $|Rsk|$ increase for t = 0, 0.01, 0.05, 0.1 and 0.2. It can be seen that the normalized Rq_{base} become smaller as $|Rsk|$ increases. The same trend is observed for all range of $Rq_{profile}$. These results confirm that as the degree of asymmetry of a surface increases, the roughness of the reflection surface decreases. It is important to note here that the small value of t means that the amount of the peaks or valleys is small and the reflection surface with roughness Rq_{base} is dominant in the profile.

Using the normalized Rq_{base} in **Fig. 2**, the reflection coefficients I_{coh}/I_0 of the ultrasonic wave at frequency 5MHz when reflected from rough surfaces having $|Rsk|$ = 0.25, 0.5 and 1 and t = 0.05 are calculated from equation (2) and then compared with those estimated based on Kirchhoff theory, as shown in **Fig. 3**. A clear difference in the reflection coefficient between symmetry and non-symmetry surfaces is observed in the results. However, only small differences are observed between the skewed surfaces. Nevertheless, these results could provide us with the quantitative characterization of skewed rough surface by ultrasonic method while having Rq and Rsk as controlled parameters which have never been developed before.

5. Conclusion

The ultrasonic evaluation of the skewed rough surface profile based on Johnson distribution has been demonstrated numerically. It is shown that as the skewness of the rough surface profile increases, the roughness of the reflection surface Rq_{base} decreases and thus, the reflection coefficient of the ultrasonic wave increases. Experiments to demonstrate the validity of the proposed ultrasonic evaluation will be conducted in near future.

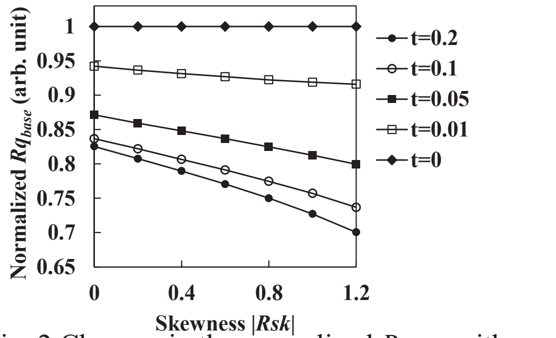

Fig. 2 Changes in the normalized Rq_{base} with $|Rsk|$ for various t.

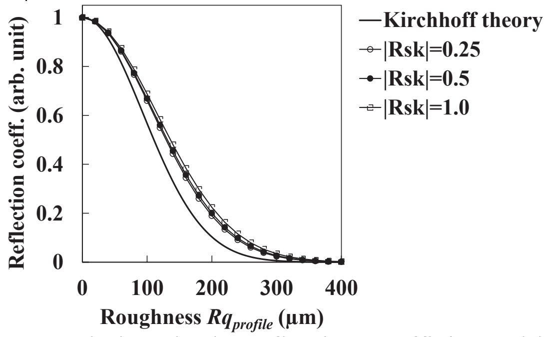

Fig. 3 Variations in the reflection coefficient with $Rq_{profile}$ for various $|Rsk|$.

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