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Estimation of Underwater Time-Reversal Channel Capacity Using Spatially Correlated Channel Signals

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1. Introduction

Since the assessement of the performance of a wireless communication system such as channel capacity requires some high-order statistics of the communication channel, an analytical expression of the capacity cannot be usually formulated. Although there have been a few studies of the topic [1], they are generally limited to some special cases because the spatial correlation cannot be completely considered. In this paper, hence, we propose a channel model to be able to generate the spatailly correlated signals. Based on the model, the time-reversal channel capacity is simulated.

2. Generation of Spatially Correlated Underwater Channel Signals

A time-varying underwater channel signal can be expressed as

$$h(t)=\sum_{k=1}^{K}A_k\mu_k(t)\exp\left(-i\theta_k(t)\right), \tag{1}$$

where k denotes the k^{th} multipath component and K is the number of total path. A_k and μ_k are the k^{th} root-mean-square of power and the k^{th} fading amplitude modeled as a complex Gaussian process with unit variance, respectively [2]. θ_k is the mean phase delay. Since A_k and θ_k represent the actual physical path-loss and propagation path, respectively, ray-tracing models (e.g. BELLHOP) can estimate those values based on physical information such as the ocean sound-speed profile, and the geometry of underwater. The fading amplitude μ_n can be theoretically generated based on the scattering by the ocean surface [3].

If multiple transducers and hydrophones operate as shown in **Fig. 1**, total $N\times M$ channel signals should be individually generated. In practical cases, however, the transducers or receivers locate in small area. Thus, A_k and θ_k in (1) can be assumed as constants for every h_{nm}: the mean of h_{nm} is identical for all n and m. If the mean of (1) is denoted as η, the fluctuation part for the m^{th} transducer to the n^{th} hydrophone is expressed as

$$f_{nm}(t)=h_{nm}(t)-\left\langle h_{nm}(t)\right\rangle=h_{nm}(t)-\eta, \tag{2}$$

where $\langle A\rangle$ is the ensemble average of A. Let us consider $N\times1$ SIMO (Single-Input-Multi-Output) channel, then each hydrophone receives the spatially correlated signal from the single transducer. Define a column vector, $\mathbf{v}$ such that

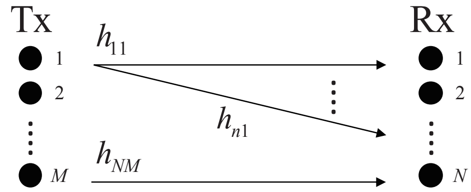

Fig. 1. MIMO (Multi-Input-Muti-Output) communication scenario.

$$\mathbf{v}=\left[f_{11}(t)\quad f_{21}(t)\quad\cdots\quad f_{N1}(t)\right]^T, \tag{3}$$

where the superscript T is the transpose operator. To generate the spatially correlated $f_{nm}(t)$, first assume that $f'_{nm}(t)$ is stochastically independent. Also, $f'_{nm}(t)$ is assumed as the complex Gaussian process with zero mean and identical variance. To obtain the spatially correlated $f_{nm}(t)$, use the matrix $\mathbf{K_g}$ defined as

$$\mathbf{K_g}=\begin{bmatrix}1 & \rho_{g12} & \cdots & \rho_{g1N}\\ \rho_{g21} & 1 & \cdots & \rho_{g2N}\\ \vdots & \vdots & \ddots & \vdots\\ \rho_{gN1} & \rho_{gN2} & \cdots & 1\end{bmatrix}. \tag{4}$$

Here, ρ_{gij} is an appropriate correlation, which can be obtained for the underwater channel by solving

$$\rho_{rij}=\frac{\left(1+\left|\rho_{gij}\right|\right)E_i\left(\dfrac{2\sqrt{\left|\rho_{gij}\right|}}{1+\left|\rho_{gij}\right|}\right)-\dfrac{\pi}{2}}{2-\dfrac{\pi}{2}}, \tag{5}$$

where $E_i(A)$ is the complete elliptic integral of the second kind with modulus A [4], and $|A|$ is the magnitude of A. ρ_{rij} is defined in terms of the fluctuating part as

$$\rho_{rij}=\frac{\mathrm{Cov}\left[\left|f_{i1}(t)\right|,\left|f_{j1}(t)\right|\right]}{\sqrt{\mathrm{Var}\left[\left|f_{i1}(t)\right|\right]\mathrm{Var}\left[\left|f_{j1}(t)\right|\right]}}, \tag{6}$$

where Cov[A,B] is the covariance of A and B and Var[A] is the variance of A [5]. In general, ρ_{rij} is known for various channels [6]. Finally, find a lower triangular matrix $\mathbf{L}$ such that $\mathbf{L}\mathbf{L}^{\mathrm{T}}=\mathbf{K_g}$. Then, the correlated vector $\mathbf{v}$ can be obtained by

$$\mathbf{v}=\mathbf{L}\mathbf{v}', \tag{7}$$

where $\mathbf{v}'$ is

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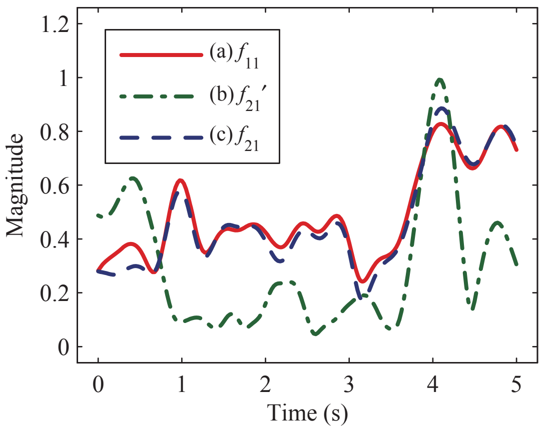

Fig. 2. Generated channel signals (a) f_{11}, (b) f_{21}', and (c) f_{21}.

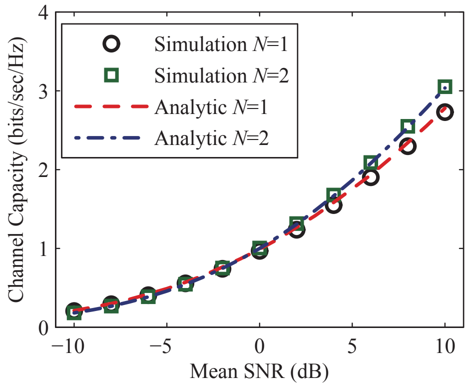

Fig. 3. Comparison between simulation results and analytic solutions for N=1 and 2.

$$\mathbf{v}' = \left[f_{11}'(t) \quad f_{21}'(t) \quad \cdots \quad f_{N1}'(t) \right]^T . \tag{8}$$

Fig. 2 shows comparison between $\mathbf{v}$ and $\mathbf{v}'$ when N and ρ_{r21} are chosen as 2 and 0.95, respectively. As can be seen, f_{21} is very similir to f_{11}, but f_{21}' is different from f_{11}.

3. Time-Reversal Channel Capacity

The formulation of the empirical channel capacity is

$$\langle C \rangle \approx \frac{1}{N_{\text{monte}}} \sum_{i=1}^{N_{\text{monte}}} \log_2 \left(1 + \frac{P_i}{P_{\text{Noise}}} \right), \tag{9}$$

where N_{monte} is the number of Monte-Carlo simulation samples. P_i is the power of the i^{th} signal sample. P_{Noise} is the noise power [7]. Considering a single transducer and N receivers, the time-reversal channel signal z can be expressed as [1]

$$z(t) = \sum_{n=1}^{N} \left| h_{n1} \right|^2 . \tag{10}$$

Thus, the power of the signal P_i in (9) is the square of (10), $z^2(t)$. To verify the proposed channel model in this paper, the analytic solution of the time-reversal channel capacity [1] is compared to the simulation result in **Fig. 3** for N = 1 and 2. For the analytical expression, the channel signals are assumed stochastically independent and have zero mean. As can be seen, the simulation results are well agreed to the analytic results. Finally, the empirical time-reversal channel capacity for the case that one transducer and 10 hydrophones are used is provided in **Fig. 4**. Each transducer is vertically distributed with the equal space of 25 cm. The horizontal range is 350 m and the depth of the transducer is 40 m. The deepest transducer is located at 50 m.

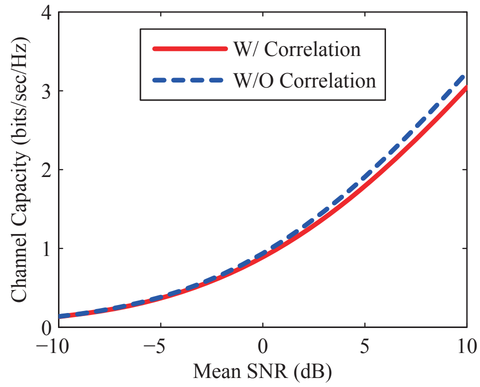

Fig. 4. Time-reversal channel capacity with and without spatial correlation.

4. Conclusion

A new MIMO channel model is proposed for the underwater channel, which can generate spatially correlated signals received by a hydrophone array. The proposed model is verified to compare the Monte-Carlo simulation and analytical results of the time-reversal channel capacity.

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