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Reflective boundary condition with arbitrary boundary shape for compact explicit-finite difference time domain method

CE-FDTD 法における任意形状の反射境界

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1. Introduction

The compact explicit-finite difference time domain (CE-FDTD) method [1,2] is suitable for the three-dimensional sound field analysis because of the high computational efficiency and the high numerical accuracy. For the practical analysis, the treatment of the reflective boundary condition with an arbitrary boundary shape is indispensable for the sound field analysis. In this study, the arbitrary boundary shape is implemented in the CE-FDTD method. While the arbitrary boundary shape have already reported for the CE-FDTD method [1], the programming is too complicated because every individual shape should be coded. In this study, we develop a simple boundary condition which is the extention of the standard FDTD method.

2. Boundary condition

In the CE-FDTD method, the arbitrary boundary shape is represented by the existence of the nodes. To calculate the sound pressure at the nodes on the boundary, the sound pressures at the nodes outside of the domain are required, but they are not defined. Therefore, it is necessary to estimate the sound pressures at the nodes outside of the domain by applying the boundary condition.

For example, as shown in Fig. 1, we here consider the calculation of the sound pressure $p_{i.j.k}^{n+1}$ at the node located on the edge of the domain indicated as the mark ●. In the figure, the black points indicated as the mark ▲ are defined in the domain, while the white points indicated as the ○,□ and ☆ are not defined. Therefore, the sound pressures at the outside nodes are estimated by using the boundary conditions.

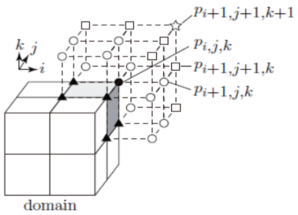

Fig. 1　Node on the boundary.

at the outside nodes are estimated by using the boundary conditions. The boundary condition differs according to the node position. So, each case is described below.

2.1 Sound pressure estimation for axially adjacent node

First, we consider the sound pressure estimation of the nodes as shown by the mark ○ in Fig. 1. These nodes are axially adjacent to the domain. The impedance boundary condition is applied to estimate the sound pressure as well as the standard FDTD method.

For an example, we consider the estimation of $p_{i+1,j,k}^{n+1}$ in Fig. 1. On the impedance boundary, the following equation can be applied [2]

$$\frac{\partial p}{\partial t} = -\bar{Z}c_0\frac{\partial p}{\partial x} \qquad (1)$$

where $\bar{Z} = Z/(\rho_0 c_0)$ is the normalized impedance of the boundary, Z is the acoustic impedance of the boundary, and ρ_0 is the density. Evaluating eq. (1) at the point $\left(i+\frac{1}{2},j,k\right)$ and the time $t = n+\frac{1}{2}$, we obtain following discretized equation;

$$\frac{p_{i+\frac{1}{2},j,k}^{n+1} - p_{i+\frac{1}{2},j,k}^{n}}{\Delta t} = -\bar{Z}c_0\frac{p_{i+1,j,k}^{n+\frac{1}{2}} - p_{i,j,k}^{n+\frac{1}{2}}}{\Delta} \qquad (2)$$

However, such as $p_{i+\frac{1}{2},j,k}^{n+1}$ or $p_{i+1,j,k}^{n+\frac{1}{2}}$ are not defined in the CE-FDTD method, so they are approximately calculated by the average between the neighboring nodes or the time steps. As the result, $p_{i+1,j,k}^{n+1}$ is obtained as

$$p_{i+1,j,k}^{n+1} = p_{i,j,k}^{n} + \alpha\left(p_{i,j,k}^{n+1} - p_{i+1,j,k}^{n}\right) \qquad (3)$$

where α is defined as follows;

$$\alpha = \frac{\bar{Z}\chi - 1}{\bar{Z}\chi + 1} \qquad (4)$$

and $\chi\ (= C_0\Delta t/\Delta)$ is the CFL number.

2.2 Sound pressure estimation for the node in a face diagonal direction

Next, we consider the estimation of the sound pressure at the nodes as shown by the mark □ in Fig. 1, which are adjacent to the domain in a face diagonal direction. On the edge boundary, the impedance relation can be written as

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$$\frac{\partial p}{\partial t} = -\frac{\bar{Z}c_0}{\sqrt{2}}\left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y}\right) \quad (5)$$

Applying the same discretization manner as axial direction to eq. (5), $p^{n+1}_{i+1,j+1,k}$ is obtained as

$$p^{n+1}_{i+1,j+1,k} = p^{n}_{i,j,k} + \frac{1}{\sqrt{2}\bar{Z}\chi + 1}\left(p^{n}_{i+1,j,k} + p^{n}_{i,j+1,k} - p^{n+1}_{i+1,j,k} - p^{n+1}_{i,j+1,k}\right) + \frac{\sqrt{2}\bar{Z}\chi - 1}{\sqrt{2}\bar{Z}\chi + 1}\left(p^{n+1}_{i,j,k} - p^{n}_{i+1,j+1,k}\right) \quad (6)$$

The same estimation method can be applied to the nodes in a space diagonal direction.

2.3 Sound pressure estimation for the corner node

Finally, we consider the sound estimation pressure on the corner node, which is adjacent to the domain in two or three directions. For an example, we consider the estimation of $p^{n+1}_{i+1,j+1,k}$ as shown in Fig. 2. This node is adjacent to the node $(i+1, j, k)$ in the *x*-direction and the node $(i, j+1, k)$ in the *y*-direction. So, two estimated values on the basis of $p_{i+1,j,k}$ and $p_{i,j+1,k}$ exist as follows

$$p^{n+1}_{(x),i+1,j+1,k} = p^{n}_{i,j+1,k} + \alpha\left(p^{n+1}_{i,j+1,k} - p^{n}_{i+1,j+1,k}\right) \quad (7)$$

$$p^{n+1}_{(y),i+1,j+1,k} = p^{n}_{i+1,j,k} + \alpha\left(p^{n+1}_{i+1,j,k} - p^{n}_{i+1,j+1,k}\right) \quad (8)$$

Therefore, the average of these pressures is applied as

$$p^{n+1}_{i+1,j+1,k} \approx \frac{\left(p^{n+1}_{(x),i+1,j+1,k} + p^{n+1}_{(y),i+1,j+1,k}\right)}{2} \quad (9)$$

The same estimation method can be applied to the corner node which is adjacent to the domain in *x*, *y* and *z* directions.

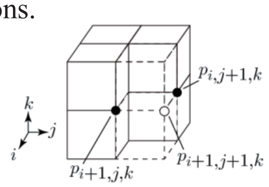

Fig. 2 Node on the concave boundary.

3. Numerical Experiments

For numerical experiments, the sound pressure in a rectangular room (4.25 × 8.5 × 2.125 m^3) is calculated. The reflection coefficient of the boundaries are set as R = 0.9. The medium is assumed to be air ($c_0 = 340$ m/s), $\chi = 0.98, \Delta = 8.5$ mm and $\Delta t = 24.5$ µs . Therefore, the numerical model consists of 1000 × 500 × 250 nodes. A sine wave of one cycle with 490µs is radiated from the point sources located at the chamber center.

Fig. 3 (a) and (b) are sound pressure distributions when $t = 24.5$ ms . Fig. 3 (a) shows the distribution of original model, and (b) shows the 45° rotated model. Both results show good agreement.

Fig. 4 is a calculated sound pressure waveform at (2.125, 2.125, 1.063) m. The solid line in the figure indicates the result of the original model corresponding to Fig. 3(a) and the dashed line is the result with the 45° rotated model corresponding to Fig. 3(b). Both results show good agreement, so the boundary conditions are implemented properly.

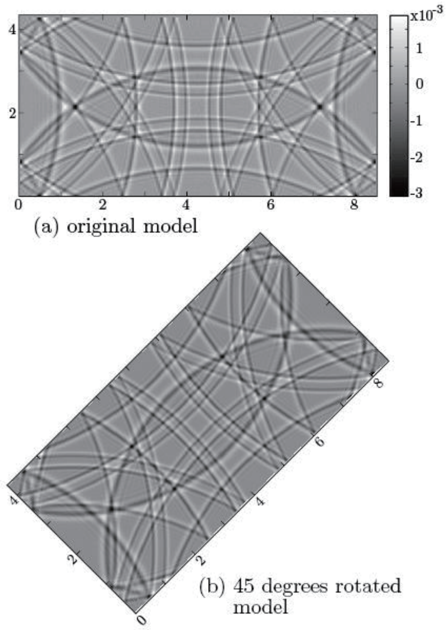

Fig. 3 Sound pressure distributions for original model and its 45° rotated model.

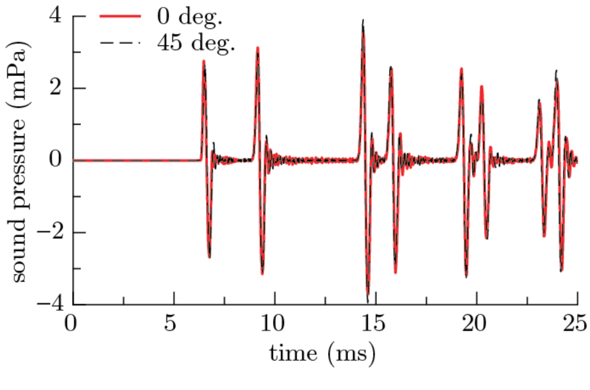

Fig. 4 Sound pressure waveforms.

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