

# Numerical Analysis of Sound Wave Propagation Using Hybrid MM-MOC Method with Non-uniform Grid

不均一グリッドを用いたハイブリッド MM-MOC 法による音響数値シミュレーション

Yuta Matsumura<sup>1†</sup>, Kan Okubo<sup>1</sup>, Norio Tagawa<sup>1</sup>, Takao Tsuchiya<sup>2</sup>, Takashi Ishizuka<sup>3</sup>

(<sup>1</sup>Facult. System Design, Tokyo Met. Univ.; <sup>2</sup>Facult. Sci. Eng., Doshisha Univ.;

<sup>3</sup>Shimizu Institute of Technology, Shimizu Corporation)

松村 勇汰<sup>1†</sup>, 大久保 寛<sup>1</sup>, 田川 憲男<sup>1</sup>, 土屋 隆生<sup>2</sup>, 石塚 崇<sup>3</sup> (<sup>1</sup>首都大 システムデザイン,<sup>2</sup>同志社大 理工,<sup>3</sup>清水建設技術研究所)

## 1. Introduction

To date, as a result of computer development, numerical analysis for sound wave propagation in time-domain has been investigated widely. The development of accurate numerical schemes is an important issue[1].

In this study, we examine the methods of characteristics (MOCs) [2] using the collocated grid as a numerical analysis method. These methods have an advantage that the treatment of the interface of different media is simpler than the staggered grid-based methods.

The constrained interpolation profile (CIP) method [3-8], a method of characteristics (MOC), is a novel low-dispersive numerical scheme. In our past study, we have applied the CIP method to numerical analyses of sound wave propagation.

New grid systems are required for the CIP large-scale simulations of wave propagation. In the previous study, sub-grid techniques [9] are proposed for the CIP method to reduce the calculation time and memory usage. However, handling the derivatives of the perpendicular directions at the interface between different sizes of grid is complicated in that technique.

Therefore, we introduced the non-uniform grid for the CIP method. This technique as well as sub-grid has an advantage of using a small amount of memory. Additionally the acoustic numerical analysis by MOCs, including CIP method, requires to set the absorbing-boundary condition (ABC), because the so-called automatic absorbing boundary (without additional outer boundary treatment) does not exhibit high-efficiency absorbing performance for multidimensional analysis. Consequently, we introduce the perfectly matched layer (PML) [10] technique into the non-uniform grid system for hybrid MM-MOC simulations of wave propagation.

## 2. Non-uniform grid system in CIP-MOC method

The governing equations for linear acoustic

fields are given in Eq.(1)

$$\nabla \cdot \vec{u} = -\frac{1}{K} \frac{\partial p}{\partial t}, \quad \rho \frac{\partial \vec{u}}{\partial t} = -\nabla p \quad (1)$$

In these equations,  $\rho$  denotes the density of the medium,  $K$  is the bulk modulus,  $P$  is the sound pressure, and  $\mathbf{U}$  is the particle velocity. Here, we assume that the calculation is for a lossless medium. In CIP analysis, these equations are transformed into advection forms.

Figure 1 depicts the schematic of a non-uniform grid model with PML, where  $L$  is the number of layers in the PML region.

In this study, we use the MM(3,1)-MOC (or, CIP method) and the MM(7,1)-MOC schemes. The MM(7,1)-MOC method employs 7<sup>th</sup>-order Hermite interpolation with four stencils for the advection calculation. Hybrid MM-MOC method is the scheme combined the MM(3,1)-MOC and MM(7,1)-MOC. That is, the advection calculation of the fine grid uses the MM(3,1)-MOC, while that of the course grid uses the MM(7,1)-MOC.

## 3. Simulation results and discussions

Figure 2 shows the geometry of the calculation model. Calculation parameters used in the calculations are summarized in table 1.

We present numerical results obtained using the non-uniform grid technique for MM(7,1)-MOC analysis. Figure 3 shows the sound pressure distribution obtained by hybrid MM(7,1)-MOC analysis with non-uniform grids. The input pressure is driven from region of the non-uniform grids. We can ascertain the propagation behavior including.

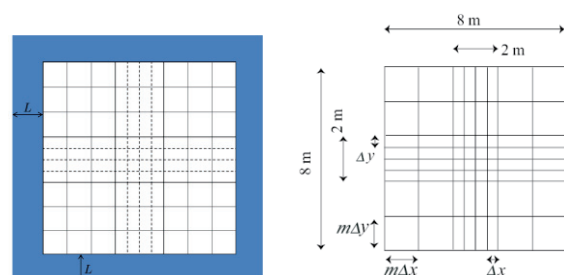


Fig. 1 Grid model with PML Fig. 2 Calculation model

Table 1 calculation parameters used in analysis

$\Delta x$	x-direction's grid width	0.01 m
$\Delta y$	y-direction's grid width	0.01 m
$\Delta t$	The discrete time width	$2.0 \times 10^{-2}$ ms
$c$	Sound velocity	340 m/s
$L$	The number of PML layer	32
$Z$	The characteristic impedance	415.03 Pa·kg/m <sup>3</sup>
	Analysis domain	8 m × 8 m
	Non-uniform grid domain	2 m × 2 m

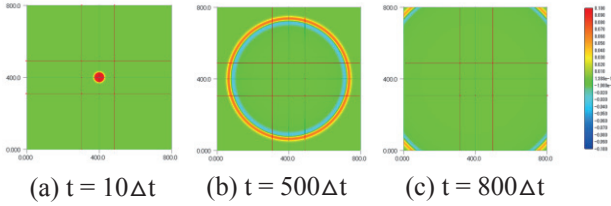


Fig. 3 Analysis of Hybrid MM-MOC method

the non-uniform grid region and find little reflection waves from PML boundaries.

Figure 4 evaluates the error using non-uniform grid by means of comparison of the absolute pressure value at some points  $((x,y) = (4.5[m],4.5[m]),(4.0[m],1.8[m]),(1.8[m],1.8[m]))$ .

The blue solid line indicates the sound pressure using non-uniform ( $m=1$ ) grid ( $P^{(r)}$ ), and the red dashed line shows the difference between  $P^{(3,1)}$  and  $P^{(r)}$  ( $|P^{(3,1)}-P^{(r)}|$ ) and the purple short-dashed line shows the difference between  $P^{(7,1)}$  and  $P^{(r)}$  ( $|P^{(7,1)}-P^{(r)}|$ ), where  $P^{(r)}$  is sound pressure using uniform-fine grid (i.e.,  $m=1$ ) as a reference.

As a result, we also find the boundary in the non-uniform grids has good permeability characteristics with an extremely low reflection.

We also investigated the calculation time required for some non-uniform grid models. Here, we used a PC with Intel Core i7-980X Extreme Edition 3.33GHz. This processor has 6 cores and 12 hyperthreaded cores, or effectively scales 12 threads. Table 2 shows the comparison results of the calculation times using hybrid MM(7,1)-MOC method.

#### 4. Conclusions

Using the hybrid MM-MOC methods, we assessed non-uniform grid systems for the MOC numerical simulation of sound wave propagation. Examinations reveal that the correct treatment of the interface between the course grids and non-uniform grids causes extremely low reflection from the boundaries. The use of a suitable non-uniform grid reduces the time and memory necessary for calculation. We also examine the PML absorbing boundary condition for the CIP-MOC 2D simulation using non-uniform grid system in this study. From the numerical results,

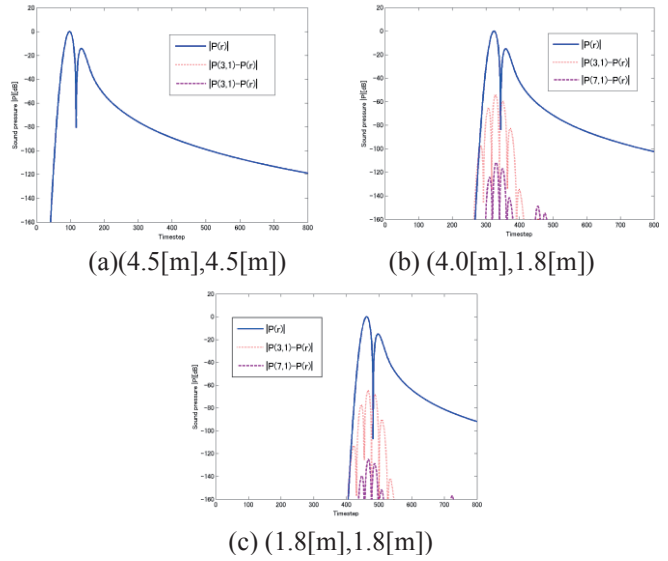


Fig. 4 The numerical error at each point( $m=5$ )

Table 2 The calculation times

Ratio of grids	Calculation time [s] (relative)
$m = 1$	473.467 (1.00)
$m = 2$	182.441 (0.38)
$m = 3$	117.596 (0.25)
$m = 4$	92.759 (0.19)
$m = 5$	79.718 (0.17)
$m = 6$	69.162 (0.14)
$m = 10$	53.332 (0.11)

PML implementation can be an effective method for non-uniform grid system with the hybrid MM-MOC methods,

#### References

1. T.Tsuchiya and A.Kumagai, Jpn. J. Appl. Phys. Vol. 48, 2009, 07GN02.
2. G.D. Smith, Numerical Solution of Partial Differential Equations, Oxford University Press, 1965.
3. T. Yabe et al.: J. of Comput. Phys., vol.169, pp.556-593, 2001.
4. K. Okubo and N. Takeuchi:IEEE Trans. Antennas Propag., vol. 55, No. 1, pp.111-119, Jan. 2007.
5. K. Okubo, S. Oh, T. Tsuchiya and N. Takeuchi: IEICE Trans. Fundamentals, vol. E90-A, No. 9, pp.2000-2005, Sept. 2007.
6. M. Konno, K. Okubo, T. Tsuchiya, and N. Tagawa,: Jpn. J. Appl. Phys., 48 (2009) 07GN01
7. T. Tsuchiya, K. Okubo and N. Takeuchi: Jpn. J. Appl. Phys., 47 (2008) pp. 3952-3958
8. M. Konno, K. Okubo, T. Tsuchiya and N. Tagawa: Jpn. J. Appl. Phys., 47 (2008), pp. 3962-3963
9. M.W. Chevalier and R.J. Luebbers, IEEE Tran. Antenn. Prop., vol. 45, 1997, pp. 411.
10. J.P.Berenger, "A perfectly matched layer for the absorption of electromagnetic waves", J.Comput. Phys., 114, 185-200(1994).