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Proceedings of Symposium on Ultrasonic Electronics, Vol. 35 (2014) pp. 465-466
3-5 December, 2014

Effects of inner tube-diameter on a coaxial thermoacoustic engine

同軸型熱音響エンジンの内管内径の影響

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1. Introduction

A thermoacoustic engine can generate acoustic waves (work flow) from industrial waste heat without moving parts. The energy conversion occurs in a narrow tube. An arrangement of many narrow tubes is called a stack.

The phase difference ϕ between the pressure P and the particle velocity U is one barometer of the contribution to energy conversion in the stack. Reversible energy conversion is realized in a stack if the phase difference on the stack (the phase of the traveling wave) is zero. Yazaki *et al*. developed a loop tube type thermoacoustic engine that realizes reversible energy conversion[1]. High-fabrication accuracy that includes consideration of heat expansion is necessary to prevent air leaks in the loop tube. A coaxial tube type acoustic cooler is described[2]. Air leakage attributable to the strain of heat expansion is suppressed in the coaxial tube. Moreover, the coaxial tube contributes to the size reduction of the thermoacoustic engine. A previous study demonstrated acoustic oscillation in a prototype coaxial thermoacoustic engine[3] with a distinct toric path (annular path) in the coaxial tube. The cross-sectional ratio of the annular path and the inner tube are related to the phase difference distribution in the tube because an acoustic wave is reflected when it propagates in the tube with different cross-sections. The reflection increases the standing wave component. Subsequently, the increase of the standing wave component reduces the heat efficiency. The low-reflection path should be constructed in the coaxial thermoacoustic engine for efficiency. It is therefore desired that the cross sectional ratio of the annular path and the inner tube be one. Here we studied the relation between the cross-sectional ratio and phase difference distribution for efficiency of the coaxial thermoacoustic engine.

2. Acoustic field analysis

Distributions of P and U on a tube axial direction are calculated using the Rott's equation[4]. The diagonalized Rott's equation is shown Eq. (1) when the temperature gradient (dT_m/dx) on the tube axial direction is zero.

$$\begin{bmatrix} P(x) \\ U(x) \end{bmatrix} = \begin{bmatrix} \cos kx & -\dfrac{i\omega\rho_\mathrm{m}}{k(1-\chi_\mathrm{v})}\sin kx \\ \dfrac{i\omega\rho_\mathrm{m}}{k(1-\chi_\mathrm{v})}\sin kx & \cos kx \end{bmatrix} \begin{bmatrix} P(0) \\ U(0) \end{bmatrix}, \qquad (1)$$

Therein, k, ω, r_m, P_m, γ, σ, and T_m respectively represent the complex wave number, the angular frequency of oscillation, the mean density, the mean pressure, the ratio of specific heats, the Prandtl number, and the mean temperature of the fluid in the tube. And, χ_v is a complex function that enables us to describe three-dimensional phenomena occurring in the tube using two one-dimensional equations.

The annular path is hypothesized as a parallel plate path if the viscous boundary layer thickness δ_v is much smaller than half the difference between two wall surfaces in the annular path[5]. Therefore, χ_v of a parallel plate path is defined as Eq. (2):

$$\chi_\mathrm{v} = \frac{\tanh\{(1+i)r_0/\delta_\mathrm{v}\}}{(1+i)r_0/\delta_\mathrm{v}}, \qquad (2)$$

where r_0 is half the difference between two wall surfaces in the parallel plate path.

Distributions of P and U in the annular path are calculated from two complex pressures $P(0)$ and $P(x)$ using Eq. (1).

3. Experimental method

A diagram of the coaxial thermoacoustic prime mover is presented in **Fig. 1**. The system fluid is air at atmospheric pressure. The outer tube is a resonant tube with both closed ends. The respective total lengths and the inner diameter of the outer tube were 2100 mm and 96 mm. A tube axial coordinate with the left end of the outer tube as $x = 0$ mm is defined. The inner tube was a resonant tube with both open ends. The range of the annular path was $50 \le x \le 2050$ mm. The outer diameter of the inner tube was changed from 13 mm to 61 mm. The stack was a 50-mm-long honeycomb ceramic with a channel having 0.45 mm radius. The low-temperature part of the stack was

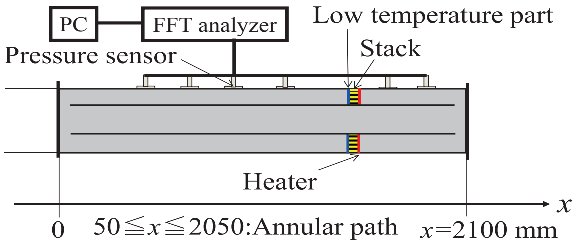

Fig. 1 Coaxial thermoacoustic engine.

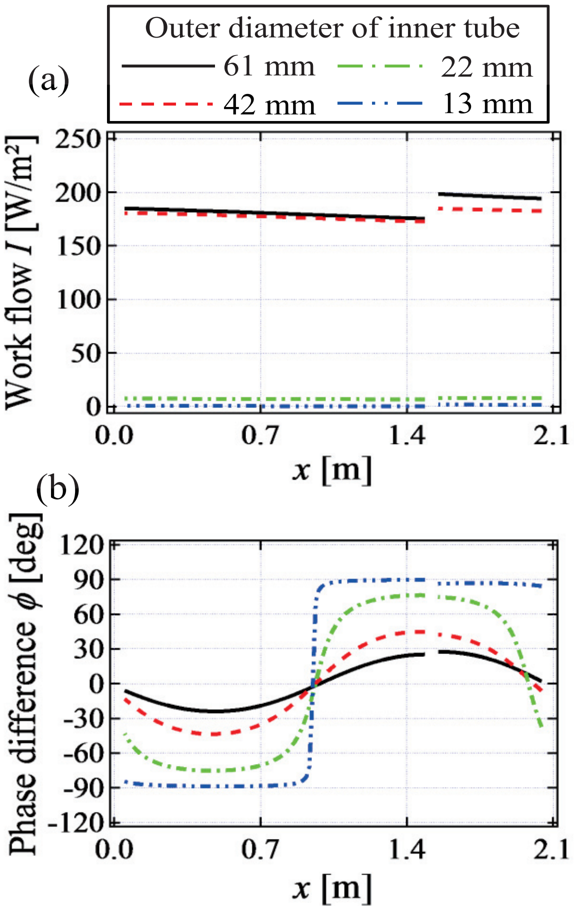

Fig. 2 Acoustic field distributions in the annular path. (a) Work flow I. (b) Phase difference ϕ between the pressure P and the particle velocity U.

installed x =1500 mm in the annular path. An electric heater was set up at the high-temperature part of the stack. Pressure sensors were set on the outer tube wall to measure the pressure in the annular path. Thermoacoustical oscillation was observed when electrical power of 330 W was supplied to the heater in each condition.

4. Experimental results

Figure 2 depicts distributions of Work flow I ($=|P||U|\cos\phi/2$) and ϕ. In Fig. 2(a), amplification amounts of I in the stack were 2, 1, 12, and 23 W/m^2 when the outer diameters of the inner tube were, respectively, 13, 22, 42, and 61 mm. In Fig. 2(b), the peak values of ϕ on the annular path were, respectively, 90, 75, 44, and 28 degree for 13, 22, 42, and 61 mm. Wall-thicknesses of inner tube were, respectively, 1, 1, 1 and 2 mm for 13, 22, 42, and 61 mm.

5. Discussion

The acoustic waves in the tube have components of traveling and standing wave. The distribution of ϕ indicates the ratio of these components. The peak value of ϕ approaches 90 degree with the increase of the standing wave component. The peak value of ϕ equals to zero if the standing wave component is zero. The standing wave component increase by the reflection when the acoustic wave propagates in the tube with different cross-sections. The peak value of ϕ approaches 90 degree as the increasing difference of cross sections.

For the coaxial thermoacoustic engine of this study, the cross section area of the annular path equals to the inner path in which the diameters of the outer and the inner tube are, respectively, 96 mm and 67 mm. In Fig. 2(b), it is therefore considered that the peak value of ϕ decreased in the inner tube diameter approaching 67 mm. Also, ϕ of the stack approaches zero as the decrease of the peak value. It is therefore confirmed that amplification amount of work flow is largest when the inner tube's outer diameter is 61 mm.

6. Conclusion

The relation between phase difference distribution and the cross sectional ratio of annular and inner tube paths is studied for efficiency of the coaxial thermoacoustic engine. The results confirmed that the excited acoustic wave conforms more closely to a traveling wave if the difference of cross sections decreases.

Acknowledgments

This research was partially supported by a Grant-in-Aid for Young Scientists (a), a Grant-in-Aid for Challenging Exploratory Research, and the Program for Fostering Regional Innovation.

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