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Robust contrast source inversion method for waveform reconstruction of the breast tissue in ultrasound computed tomography

ロバストなコントラストソースインバージョン法を用いた乳房超音波 CT アルゴリズム

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1. Introduction

Ultrasound computed tomography (USCT) for the breast cancer diagnosis has recently been developed resulting from the diagnostic device equipped with the novel ring-array transducers and the rapid improvement of computing hardware. Several works have focused on the ray-based reconstruction for the sound-speed image of the breast tissue, yielding a low-resolution and blurring result due to ignoring the diffraction effects; see [1]. In this study, the contrast source inversion (CSI) method is introduced to overcome these difficulties by exploiting the wave theory in the frequency domain. And hence, it produces a high-accuracy image of the breast tissue in comparison with the ray-based reconstruction, and besides a relatively low computing cost compared with the waveform inversion in time domain.

The motivation of the CSI method is to linearize the original nonlinear waveform inversion model by adding a new variable, namely, a contrast source. Then we solve a two-objective optimization problem for the sound speed. In terms of the Robust CSI method proposed in [2], the total-variation (TV) regularization method is additionally chosen to diminish the error of the data fidelity terms caused by the noise data. Moreover, for the demand of breast cancer diagnosis, TV regularization is preferred since it preserves more edge information in the reconstructed image.

However, we will face several tasks while utilizing the methodology of regularization methods with the conventional view: 1) determine the unknown noise level in practice; 2) determine the regularization parameter prior to executing the algorithm. But in this study, one can heuristically use the automatic choice of regularization parameters without a priori information of the noise level.

2. Mathematical Formulation and Algorithms

The Lippmann-Schwinger equation, governing the acoustic-wave propagation of the USCT problem, gives the total field of pressure wave by the integral equation in the frequency domain

$$u_i(x) - u_i^{\mathrm{inc}}(x) = k^2 \int_{\mathbb{R}^2} G(x,y) f(y) u_i(y) dy, \quad (1)$$

where $i = 1, \cdots, N$ indicates the source number, $u_i(x)$ represents the total wave field ($x \in \mathbb{R}^2$), $G(x,y)$ the 2-dimensional free-space Green's function of homogeneous Helmholtz equation with the wave number k, $u_i^{\mathrm{inc}}(x)$ the spherical incident wave field with respect to ith source and $f(x)$ the refraction index with a compact support

$$f(x) = \frac{1}{c^2(x)} - \frac{1}{c_0^2}$$

where $c(x)$ and c_0 are the sound speed of the breast tissue and the homogeneous background. The USCT problem is to recover $f(x)$ in D from the Lippmann-Schwinger equation **(1)** given the data $u_i(s)$ on the boundary S, corresponding to the incident wave $u_i^{\mathrm{inc}}(x)$ in $\mathbb{R}^2$.

To solve this nonlinear inverse problem, one can linearize it with the help of an auxiliary function -- contrast source $v_i(x) = u_i(x) f(x)$, which yields the data equation

$$g_i(s) = u_i(s) - u_i^{\mathrm{inc}}(s) = G^S v_i(s), s \in S \quad (2)$$

and the state equation

$$v_i(x) - f(x) u_i^{\mathrm{inc}}(x) = f(x) G^D v_i(x), x \in D. \quad (3)$$

The robust CSI method here is introduced to solve these two equations **(2)** and **(3)** together for the unknown refraction index $f(x)$ and the contrast source $v_i(x)$ in D by minimizing the following cost functional

$$J(f, \bar{\boldsymbol{v}}) = \mu_S \sum_{i=1}^{N} ||G^S v_i - g_i||_{L^2(D)}^2 + \mu_D \left(\sum_{i=1}^{N} ||f G^D v_i - v_i + f u_i^{\mathrm{inc}}||_{L^2(D)}^2 + \lambda ||\nabla f||_{L^1(D)} \right) \quad (4)$$

with the contraction coefficients

$$\mu_S = \left(\sum_{i=1}^{N} ||g_i||_{L^2(D)}^2 \right)^{-1}, \quad (5)$$

$$\mu_D = \left(\sum_{i=1}^{N} ||f u_i^{\mathrm{inc}}||_{L^2(D)}^2 \right)^{-1} \quad (6)$$

and the regularization parameter λ.

Next, we construct a specific algorithm for the robust CSI method of **(4)**. For the outer iteration step, an alternative way to switching the iterating directions between f and v_i is the well-known alternating minimization method [3]; see **Algorithm 1**.

Algorithm 1 Alternating Minimization Method

Input $f_0, \lambda_0, v_{0,m}$, for $m = 1, \cdots, N$.

For $k = 0, 1, \cdots, k_{max} - 1$

Compute μ_S and μ_D with **(5)** and **(6).**

$v_{k+1,m} = \text{argmin}_{v_m} \mu_S \sum_{i=1}^{N} ||G^S v_i - g_i||^2_{L^2(S)} + \mu_D \sum_{i=1}^{N} ||f_k\, G^D v_i - v_i + f_k u_i^{\text{inc}}||^2_{L^2(D)} := \text{argmin}_{v_m} V(v_1, \cdots, v_N, f_k)$ for each $m = 1, 2, \cdots, N$.

Update λ_{k+1} automatically from λ_k.

$f_{k+1} = \text{argmin}_f \sum_{i=1}^{N} ||f\, G^D v_{k+1,i} - v_{k+1,i} + f u_i^{\text{inc}}||^2_{L^2(D)} + \lambda_{k+1} ||\nabla f||_{L^1(D)} := \text{argmin}_f F\left(v_{k+1,1}, \cdots, v_{k+1,N}, f, \lambda_{k+1}\right)$.

end

Output $f_{k_{max}}$.

The inner iteration of algorithm is directly given by the common numerical methods. In specific, $v_{k+1,i}$ can be updated by the conjugate gradient method and f_{k+1} by the lagged diffusivity method for TV regularization of image denoising [4].

3. Automatic Choice of the Regularization Parameters

In this section, we complete the choice rule of regularization parameters in the above alternating minimization method. Briefly speaking, we state a fixed point algorithm for resolving the regularization parameters while iterating, originally proposed in [5]. This algorithm is based on a balancing principle and the model function approach. Not only they make the numerical scheme concise, the regularization parameters also converge faster than the main iterations of $v_{k,i}$ and f_k, resulting in a stably decreasing residuals and hence a robust algorithm.

The idea of the balancing principle is to balance the data fidelity term $||fG^D v_i - v_i + f u_i^{\text{inc}}||^2_{L^2(D)}$ and the penalty term $\lambda||\nabla f||_{L^1(D)}$ of the cost functional $F(\lambda) := F(v_1, \cdots, v_N, f, \lambda)$. Its derivative is the penalty term $F'(\lambda) = \lambda||\nabla f||_{L^1(D)}$ from Lemma 2.2 in [6]. Then the optimal regularization parameter $\lambda^\dagger$ should satisfy the following equation

$$F(\lambda^\dagger) = \sigma\left(F(\lambda^\dagger) - \lambda^\dagger F'(\lambda^\dagger)\right). \quad (7)$$

To fast compute it by the method of model function approach, we use a rational function $m(\lambda)$ to approximate the same asymptotic behavior as $F(\lambda)$

$$m(\lambda) = b + \frac{c_1}{c_2 + \lambda}. \quad (8)$$

The determination of the coefficients is finally demonstrated by the fixed point iteration in *k*th step

$$F(\lambda_{k+1}) = \sigma\left(F(\lambda_k) - \lambda_k F'(\lambda_k)\right). \quad (9)$$

4. Result of Numerical Experiment

In this section, we present an example from an MRI medical phantom in **Fig 1 (a)**. We generate the synthetic data by the forward solver in [7], and run the robust CSI with both fixed regularization parameters and the dynamic ones. The relative error and the cross section of sound-speed distribution are presented to illustrate the convergence and the robustness of our proposed method.

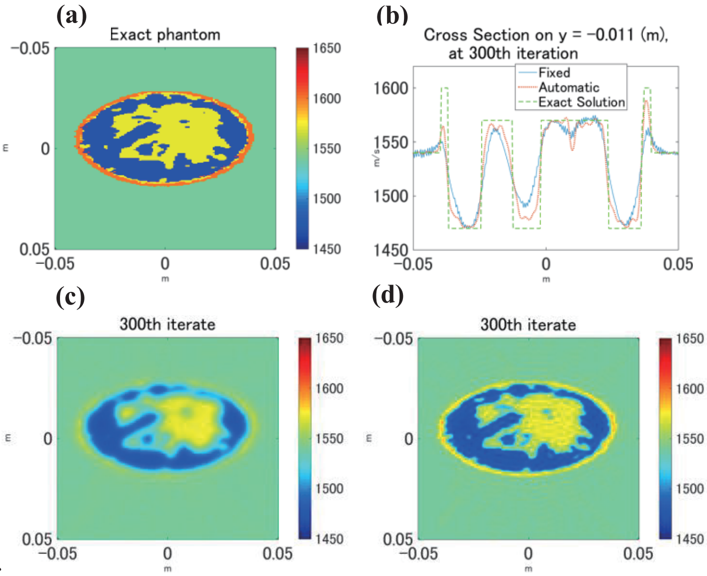

Fig 1 (a) an exact sound-speed image; **(b)** the cross section on *y=-0.011* at *300*th iterate; **(c)** and **(d)** CSI method with fixed and automatic choice of the regularization parameters.

At 300th iterate for each choice, **Fig 1 (c)** shows a blurred sound-speed image by a fixed $\lambda = 2 \times 10^{-4}$, while **Fig 1 (d)** illustrates a sharper boundary by automatic choice starting at $\lambda = 2 \times 10^{-4}$. And **Fig 1 (b)** shows the automatic strategy converges faster to the exact cross section than the fixed one.

5. Conclusion

A robust CSI method with TV regularization is presented for solving the waveform USCT problem. And an automatic choice of the regularization parameters is introduced so as to determine them efficiently without a priori information of the noise level and improve the quality of the image considered practically.

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