1P1-5

Proceedings of Symposium on Ultrasonic Electronics, Vol. 36 (2015)
5-7 November, 2015

Disadvantage of classical and quantum theories in treatment of electromechanical coupling system and advantage of complex series dynamics

電気機械結合系の扱いの古典及び量子論的問題点並びに複素級数力学の利点

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1. Introduction

In the field of engineering, the analysis in the frequency domain has been widely adopted in a lumped-parameter basis. For example, the characteristics of the electromechanical coupling system can be analyzed by calculating the relationship between electrical resonance frequencies and antiresonance frequencies on a lumped-parameter circuit. In this method, the existence of lumped-parameter dielectric capacitance components (C_0 in Mason's circuit) is inevitable. The dielectric capacitance component represents the phenomenon in which the elastic vibration interacts with the dielectric effect in the system and the acoustic speed is changed. The methodology using lumped parameters is essentially a basis of classical Newtonian mechanics.

However, some problems are revealed in the treatment of classical framework when the analysis is performed in the time domain [1]; that is, when transient or impulse response of the system is calculated. Unfavorable results are caused by the existence of dielectric capacitance and the dissipation (loss), which suggests the application limitations of classical Newtonian mechanics.

Since the quantum theory includes the classical theory within it; in other words, the framework of Newtonian mechanics can be derived from the quantum mechanics, the quantum theory should also be investigated from the viwepoint of the treatment of dielectric and dissipation phenomena.

In the analysis of electromechanical coupling systems, a method termed "complex series dynamics" [1-6], in which an interaction process between "elastic mode" and "dielectric mode" has been developed; it can treat the phenomenon on a distributed-parameter basis without any conventional lumped-parameter components.

In this study, with regard to the treatment of dielectric and dissipation phenomena, the disadvantage of the classical and quantum mechanical framework is discussed, and the improvement of the treatment using complex series dynamics is discussed from the viewpoint of the comparison with the classical and quantum mechanical framework.

2. Treatment of Dissipation Phenomenon

In quantum mechanics, the time evolution of a state vector is expressed with

$$|\psi(t)\rangle = U(t, t_0)\,|\psi(t_0)\rangle \ , \tag{1}$$

where

$$U(t, t_0) = \exp(-j\boldsymbol{H}(t-t_0)/\hbar) \tag{2}$$

in the Schrödinger picture, where t_0 is an initial time, $|\psi(t)\rangle$ is a state vector at time t, $U(t, t_0)$ is a unitary operator for the non-dissipative evolution from time t_0 to t, and $\boldsymbol{H}$ is the Hamiltonian operator. The wave function $\psi(x,t)$ is changed as

$$\psi(x,t) = \int K(x, t; x_0, t_0)\,\psi(x_0, t_0)\,dx_0 \ , \tag{3}$$

where

$$K(x, t; x_0, t_0) = \langle x\,|\,U(t, t_0)\,|\,x_0\rangle \ . \tag{4}$$

The Feynman kernel $K(x, t; x_0, t_0)$ can be calculated by the superposition of phase factor in the form of

$$\exp(jS/\hbar), \tag{5}$$

on all the possible paths in the system, where S is the time integral of Lagrangian L---the action integral.

However, when the dissipation is included in the system, the treatment of Hamiltonian H or Lagrangian L becomes difficult, which is the disadvantage of the quantum theory. The "Caldeira-Leggett model"[6] can treat the dissipation in the framework of quantum mechanics, in which some interaction with "environment" is considered by introducing interaction terms in Lagrangian. Some complicated calculation leads to the classical equation of motion with a dissipative term.

However, the classical equation of motion is described with a time differential equation of second order, and therefore, two eigenvalues (two resonance frequencies) for one spatial degree of freedom are obtained by solving the equation. As the dissipation becomes larger, an unreasonable situation for the phenomenon of wave propagation occurs, since one of the two eigenvalues approaches zero. The inconsistency caused in the framework of Newtonian mechanics suggests that the same inconsistency is potentially included in the framework of the quantum mechanics, from which the Newtonian mechanics is derived.

In the author's method (complex series dynamics), not only mechanical (elastic) but also electric (dielectric) phenomena in the electromechanical coupling system can be treated. The characteristics of the system is obtained by the superposition of energy modes with the following forms of phase factor:

$$\text{Elastic mode:}\quad \exp(-j\omega T - a_e), \tag{6}$$

$$\text{Dielectric mode:}\ \exp(-j0 - a_d) = \exp(-a_d), \tag{7}$$

where ω is the angular frequency of the elastic mode as a wave, T is propagation time for the elastic mode to pass through a spatial domain in the system, and a_e and a_d are attenuation factors of elastic and dielectric modes, respectively, on the spatial domain. The superposition is performed over all the possible paths in the system. This methodology of superposition is similar to that of Feynman's path integral.

However, in Feynman's method, when

$$L = p\ \mathrm{d}x/\mathrm{d}t - H = \hbar k\, \mathrm{d}x/\mathrm{d}t - \hbar\omega\ , \tag{8}$$

where p is the linear momentum and k is the wave number, eq. (5) becomes

$$\exp(jS/\hbar) = \exp(j\ k(x - x_0) \text{ - } j\ \omega\ (t - t_0)), \tag{9}$$

in which the quantum particle propagates from (x_0, t_0) to (x, t). For wave propagation, the amplitude and phase in eq. (7) do not change from (x_0, t_0) to (x, t).

On the other hand, in the complex series dynamics, the spatial translation is expressed with the shift of the elements of a vector. For example, for the spatial degree of freedom $N = 4$, the state is expressed with

$$\boldsymbol{\eta} = (\eta_1, \eta_2, \eta_3, \eta_4), \tag{8}$$

and the spatial translation from the domain 1 to the domain 2 is expressed conceptually as

Elastic mode:

$$(1, 0, 0, 0) \rightarrow (0, \exp(-j\omega T - a_e), 0, 0)\ , \tag{10}$$

Dielectric mode:

$$(1, 0, 0, 0) \rightarrow (0, \exp(-\,a_d), 0, 0), \tag{11}$$

when the coupling between the modes does not occur.

When the coupling occurs, the energy between the modes is exchanged with each other via an unitary matrix. The advantages of the present method over Feynman's method (eqs. (8) and (9)) are as follows:

(i) The dissipation can be described;

(ii) The energy exchange and the spatial distribution of the mode are easily calculated.

3. Treatment of Dielectric Phenomenon

In the complex series dynamics, the interaction between the elastic mode and dielectric mode is expressed with two types of unitary processes:

(i) One is termed "point interaction" in which a finite quantity of coupling between the two modes occurs on a spatial point on the system with an infinitesimal interaction length;

(ii) The other is termed "continuous interaction" in which an infinitesimal quantity of coupling between the two modes occurs in an integral manner in a finite interaction length on the system.

The combination of the two types of unitary processes can only lead to the correct calculation result corresponding to an actual physical phenomenon, while the conventional lumped-parameter-based equivalent circuit methods cannot deal with electromechanical coupling systems reasonably in the time domain.[1)] This suggests that the phenomenon is not caused on a classical lumped-parameter basis, but essentially on a distributed-parameter basis.

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