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Dispersion property of guided waves propagating a helical structure obtained by analysis and experiment

らせん構造におけるガイド波の分散特性-解析と実験による検討-

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1. Introduction

Guided wave ultrasonic modes propagating over long distances can be applied to non-destructive evaluation(NDE) for wire-ropes. Guided waves have a dispersion property, which represents frequency dependence on propagating velocities. Therefure, for effective use of guided waves, we need to know this property, which is usually expressed as dispersion curves[1]~[2]. Furusawa et al.[3] remarked L-mode of guided waves have advantages for NDE.

Dispersion curves of complex structures including wire-ropes are generally obtained by a semi-analytical finite element (SAFE) method [2],[4]~[10], because the SAFE needs a smaller amount of calculation time and memory than the finite element mehod (FEM). However, if a cross-section is a semi infinity rectangle or a circle, a full analytical method can be developed even for complex geometries including helical structures.

Thus, the purpose of this study is to obtain dispersion cuves of L-mode waves propagating a helical structure by a full analytical method. Obtained results are compared with experimental results.

2. Analysis

1. Governing equation

The Navier gorverning equations, which is the equation of motion for an elastic isotropic solid, are:

$$\rho\frac{\partial^2\boldsymbol{u}}{\partial t^2}=(\lambda+2\mu)\boldsymbol{\nabla}\boldsymbol{\nabla}\cdot\boldsymbol{u}-\mu\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{u} \quad (1)$$

This equation can also be written in the cylinderical coordinate system. Guided waves along cylindrical structures propagate as harmonic waves expressed by,

$$u_r=U(r)\cos n\theta e^{i(kz-\omega t)} \quad (2.1)$$

$$u_\theta=V(r)\sin n\theta e^{i(kz-\omega t)} \quad (2.2)$$

$$u_z=W(r)\cos n\theta e^{i(kz-\omega t)} \quad (2.3)$$

Here we consider propagation of L-mode waves along a helical structure. Now, a new coordinate sytem (r,θ,s) is constructed from the orthonormal basis $(\boldsymbol{N},\boldsymbol{B},\boldsymbol{T})$, for which a position vector $\boldsymbol{\phi}(r,\theta,s)$ can be expressed as,

$$\boldsymbol{\phi}(r,\theta,s)=\boldsymbol{R}(s)+\boldsymbol{N}(s)r\cos\theta+\boldsymbol{B}(s)r\sin\theta \quad (3)$$

Using the above Serret-Frenet formulate, it can be shown that this kind of mapping yields the following non-orthogonal covariant basis $(\frac{\partial\boldsymbol{\phi}}{\partial r},\frac{\partial\boldsymbol{\phi}}{\partial\theta},\frac{\partial\boldsymbol{\phi}}{\partial s})$, denoted by $(\boldsymbol{g_1},\boldsymbol{g_2},\boldsymbol{g_3})$. Here, κ is the curvature and τ is the tortion.

$$\boldsymbol{g_1}=\frac{\partial\boldsymbol{\phi}}{\partial r}=\boldsymbol{N}(s)\cos\theta+\boldsymbol{B}(s)\sin\theta \quad (4.1)$$

$$\boldsymbol{g_2}=\frac{\partial\boldsymbol{\phi}}{\partial\theta}=-\boldsymbol{N}(s)r\sin\theta+\boldsymbol{B}(s)r\cos\theta \quad (4.2)$$

$$\boldsymbol{g_3}=\frac{\partial\boldsymbol{\phi}}{\partial s}=\boldsymbol{T}(s)+\big(\tau\boldsymbol{B}(s)+\kappa\boldsymbol{T}(s)\big)r\cos\theta-\tau\boldsymbol{N}(s)r\sin\theta \quad (4.3)$$

The covariant metric tensor, denoted by $\boldsymbol{g_{ij}}=\boldsymbol{g_i}\cdot\boldsymbol{g_j}$, and the Christoffel symbol are given by

$$\boldsymbol{g}=\begin{bmatrix}1&0&0\\0&r^2&\tau r^2\\0&\tau r^2&(\tau r)^2+(1-\kappa r\cos\theta)^2\end{bmatrix} \quad (5)$$

$$\Gamma_{ij}^k=\boldsymbol{g_{i,j}}\cdot\boldsymbol{g^k} \quad (6)$$

where the convariant basis $(\boldsymbol{g^1},\boldsymbol{g^2},\boldsymbol{g^3})$ is defined by $\boldsymbol{\delta_j^i}=\boldsymbol{g^i}\cdot\boldsymbol{g_j}$.

The strain and stress tensors in the helical coordinate system can be described by using the displacement expressed in the cylinderical coordinate system with the Christoffel symbol as follows,

$$\epsilon_{ij}=\frac{1}{2}\big(u_{i,j}+u_{j,i}\big)-\Gamma_{ij}^k u_k \quad (7)$$

$$T_{ij}=C_{ijkl}\epsilon_{kl} \quad (8)$$

Here the boundary condition is assumed to be stress-free

$$T_{rr}|_{r=a}=0 \quad (9)$$

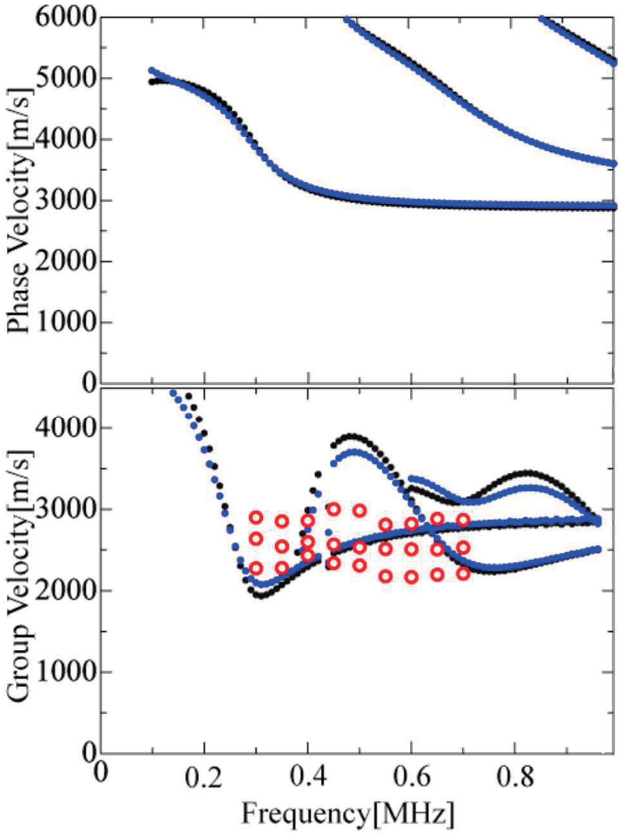

Fig: 1 Dispersion curves
(Black and blue dots indicate analytical results and red circles indicate experimental results)

Fig: 2 Experimental helical specimen

$$T_{rz}|_{r=a} = 0 \quad (10)$$

where a is the radius of the cross-section of the helical structure.

The determinant of the coefficient matrix appearing in equations (8) satisfying this boundary condition must be zero to have non-trivial solutions. This gives the dispersion equations in the helical coordinates.

2. Analytically obtained dispersion curves

Fig:1 shows analytically obtained dispersion curves of L-mode for experimental helical parameters. Black dots indicate dispersion curves for inner paths and blue dots indicate dispersion curves for outer paths in a helical volume.

3. Experiment

In our experiment, guided waves along a 1-meter aluminium helical specimen, shown in Fig: 2, were investigated with electromagnetic acoustic transducers (EMATs). The curvature of the specimen was 3.47[1/m], and its torsion was 0.897[1/m].

Experimental results are plotted by red circles in Fig: 1. They are roughly close to analytical results, though more delicate transmission and detection of waves may be required for higher accuracy.

4. Conclusions

An analytical method of calculating dispersion curves for helical structures has been developed by introducing the helical coordinate. Obtained results are roughly close to experimental results.

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