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Eigenmode Analysis of a Lamb Wave Isotropic Plate Waveguide by the Finite-Difference Time-Domain Method Using Staggered Grids with Collocated Grid Points of Velocities

SGCV を用いた FD-TD 法による等方性平板ラム波導波路の固有モード解析

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1. Introduction

The staggered grid with the collocated grid point of velocities (SGCV) was presented for the finite-difference time-domain (FD-TD) method to model propagation of elastic waves in anisotropic solids, and was applied to resonance frequency analysis of a Lamé mode resonator on an isotropic solid to demonstrate SGCV's validity and usefulness: the free boundary[1)] and symmetric boundary conditions[2)] can simply be implemented.

In acoustic wave devices, resontors and waveguides are basic elements. We have demonstrated the resonance frequency analysis of a Lamé resonator [2,3)] but have not done the eigen mode analysis of guided waves using the FD-TD method with SGCVs.

In this paper, we discuss eigenmode analysis of a Lamb wave isotropic plate waveguide by FD-TD method with SGCVs. We demonstrated that the normalized dispersion cureves can be computed with the specified Poisson ratio without the mass density and stiffness tensor, and the thickness of the isotropic plate from time response data of velocities at equally spaced observation points along the Lamb wave propagation direction.

2. FD-TD model of the Lamb wave plate waveguide

We consider a two-dimensional free isotropic plate with its thickness b. **Fig. 1** shows a numerical model: the left side of the hatched region, which is discretized with SGCVs, is symmetric boundary and the right side is truncated at $x = L$ where $L = N_l\Delta > v_n N_t \Delta_t$ because the Lamb waves can not arrive at the right side in period, $[0, N_t\Delta_t]$. Here, Δ and Δ_t are the space and time intervals, N_l and N_t are numbers of special grids of the length L and the total time steps, respectively, and v_n is the phase velocity of the P-wave. The upper, lower and truncated boundaries are free surfaces.

The normalized Newton's equation of translation motion and Hook's law are represented in a Cartesian coordinate system (x, y, z) as

$$\Delta_t \frac{\partial v_x}{\partial t} = (R\Delta)\frac{\partial T_{xx}}{\partial x} + (R\Delta)\frac{\partial T_{xz}}{\partial z},$$
$$\Delta_t \frac{\partial v_z}{\partial t} = (R\Delta)\frac{\partial T_{xz}}{\partial x} + (R\Delta)\frac{\partial T_{zz}}{\partial z},$$
$$\Delta_t \frac{\partial T_{xx}}{\partial t} = (R\Delta)\frac{\partial v_x}{\partial x} + (R\Delta\cdot\frac{\sigma}{1-\sigma})\frac{\partial v_z}{\partial z},$$
$$\Delta_t \frac{\partial T_{zz}}{\partial t} = (R\Delta)\frac{\partial v_z}{\partial z} + (R\Delta\cdot\frac{\sigma}{1-\sigma})\frac{\partial v_x}{\partial x},$$
$$\Delta_t \frac{\partial T_{xz}}{\partial t} = (R\Delta\cdot\frac{(1-2\sigma)}{2(1-\sigma)})(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}),$$

where R is the Courant number defined as $R = v_n\Delta_t/\Delta$, σ is the Poisson ratio of the plate, $v_i = \sqrt{\rho c_{11}} v_i'$ ($i = x,\ z$) is the normalized i-component of the particle velocity, v_i', with the density ρ and the stiffness c_{11}, and T_{ij} $(i, j = x, z)$ is the ij-component of the stress tensor.

We note that FD-TD equations derived from above equations using second-order accuracy scheme in the time and spacial differences can be computed with σ and R.

A vibrating force source at the point (0, $b - \Delta/2$) on the symmetric boundary is a sine modulated Gauusian pulse expressed as $\hat{x}\sin(2\pi\, n_t/m)\ \exp[-(N_t - n_0)^2/(2n^2)]$, where $\hat{x}$ is a unit vector in the x-axis direction, the center frequency, f_0, of the sine modulation is $1/(m\Delta_t)$, the discrete time is defined as $t_n = n_t\Delta_t$ $(n_t = 1,2,\cdots,N_t)$, the width and the center, w and f_0, of the Gaussian pulse are $n\Delta_t$ and $n_0\Delta_t$, respectively.

The free surface condition can be implemented by the restriction of the normal componets of stress tensors being zero and using bilinear interpolations of the velocity vectors at the points on the free surfaces. [1)] The symmetric boundary condition can be imposed after Ref.1.

Computed time responses of the values of v_x at the observation points $(\Delta(i - 1/2),\ b - \Delta/2)$ $(i = 1,2,\cdots,N_x)$ along the x-axis are recorded for extracting dispersion curves by conversion of time series data to frequency-propagation constant spectra by the two-dimensional fast Fourier transform (2D-FFT).

For computing the time responses of the plate waveguide with the Poisson ratio σ by FD-TD method, parameters of the source (m, n, and n_0)

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and grids ($N_b = b/\Delta$, N_x, N_l, and N_t) must be determined. In addition, the Courant number R must be specified.

3. FD-TD Parameter setting for mode analysis

In this section, we will discuss FD-TD patameter settings for computing a normalized dispersion curves, the $\omega b(\pi v_s)^{-1}$- βb diagram in the range $0 \le \omega b(\pi v_s)^{-1} \le \omega_N$ and $0 \le \beta b \le \beta_N$, of the Lamb wave plate waveguide with the Poisson ratio σ and SV-wave velocity v_s. The dispersion curves can be plotted by finding local peaks of the normalized frequency spectrum,$\omega b/\pi v_s$, discretized with equaly intervals, Δ_ω, at each normalized propagation constant, βb, discretized with Δ_β. The uncertanity of dispersion cureves is a rectangle $\Delta_\omega \times \Delta_\beta$ on the $\omega b(\pi v_s)^{-1}$- βb plane.

By rule of thumb for the number of grids per the maximum value of SV wave's wavelength, we may have a condition as $2N_b / \omega_N \ge 10$.

Considering the source Gaussian pulse function as zero except the time interval $[(n_0 - C_0 n)\Delta_t, (n_0 + C_0 n)\Delta_t]$, we should satisfy a condition as $n_0 > C_0 n$ and expect that the normalized frequency range of excited waves is $f_0\Delta_t - C_0(2\pi n)^{-1} \le f_0\Delta_t \le f_0\Delta_t + C_0(2\pi n)^{-1}$. Hence, we may choose the center and upper normalized frequencies as $f_0\Delta_t = m^{-1} = v_s\omega_N\Delta_t(4b)^{-1}$ and $f_0\Delta_t + C_0(2\pi n)^{-1} = v_s\omega_N\Delta_t(2b)^{-1}$, respectively. Therefore, we have following relations: $N_b m^{-1} = C_0 N_b (2\pi n)^{-1} = v_s\omega_N\Delta_t(4b)^{-1} = R\ \omega_N(v_s/4v_n)$. Recalling that $v_s/v_n = (1-2\sigma)^{1/2}(2-2\sigma)^{-1/2}$, we determine $N_b m^{-1}$ and $N_b(2\pi n)^{-1}$ with σ, R, and ω_N.

The 2D-FFT of the time response in the period of $N_t\Delta_t$ and $N_x\Delta$ has the intervals of the discretized normalized frequency and propagation constant, $\Delta_\omega = 2N_b(v_s/v_n)^{-1}(N_t R)^{-1}$ and $\Delta_\beta = 2\pi(N_b/N_x)$. Hence, we can determine N_t and N_x with σ, R, Δ_ω and Δ_β.

The truncated boundary gives the condition $L = N_l\Delta > v_n N_t \Delta_t$ to the numerical model and we have the relation, $N_l > N_t R$. Note that $N_l \ge N_x$.

4. Numerical Results

When the Poisson ratio of the plate waveguide is 0.31, $\omega_N = 9$, $C_0 = 2, \Delta_\omega = 0.02$ and $\Delta_\beta = 0.02$, we first determine the value of N_b being 2^6, then, choosing N_t and N_x as a power of two, we can determine these values: $N_x = 2^{15}$ and $N_t = 2^{15}$. Here, R=0.5. Choosing the values m, $2\pi n$ and n_0 from intergers, we have following values: m=109, $2\pi n = 2m = 218$ and n_0=105>2n. Finally, we choose the N_l value of 2^{15} for $N_l > N_t R$ and $N_l \ge N_x$. **Figure 2** shows the numerical dispersion curves computed by 2D-FFT. We remark that Fig.2 was not drawn by finding local peak algorithm because our algorithm missed mode branches. The results computed by FD-TD method (the solid lines) agree with the results[4] of the Rayleigh-Lamb frequency equations.

5. Conclusions

In this paper, dispersion relation analysis of the Lamb wave isotropic plate waveguide by the FD-TD method has been carried out. The 2D-FFT technique is used to calculate the wave field in the frequency domain. We confirm the validity of mode analysis by the FD-TD method with SGCVs. Numerical analysis of field distributions and convergence rates of propagation constants of Lamb waves, and simulation of nondestructive evaluation with Lamb waves are our future work.

References

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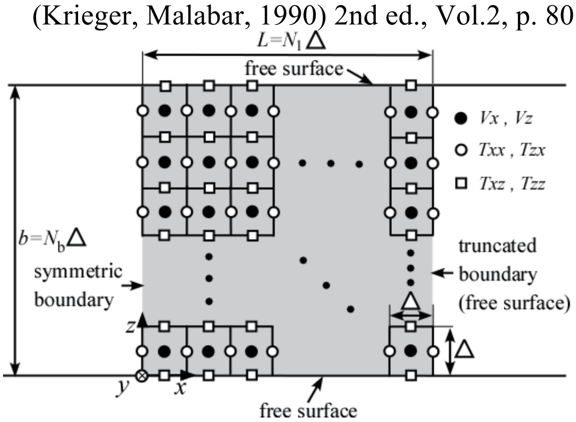

Fig.1 Numerical model of a Lamb wave plate waveguide. The hatched region models the infinite long plate.

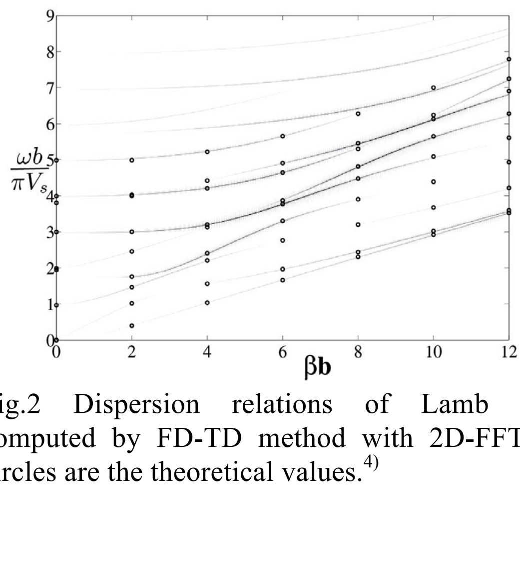

Fig.2 Dispersion relations of Lamb waves computed by FD-TD method with 2D-FFT. The circles are the theoretical values.[4]