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Numerical Analysis of Capillary Wave for Ultrasonic Nebulizing

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1. Introduction

The capillary wave caused by ultrasound on the water has been studied by many researchers[1,2]. The amplitude divergence of the resonated capillary wave has been known as an important phenomena that explains the ultrasonic nebulizing mechanism. Even though many theoretical researches have been done, which derived the relationship between the droplet sizes and driving ultrasonic frequency based on many experimental results, they still have many approximations and assumptions[3]. In this study, the characteristic analysis of ultrasonic nebulizing, which was derived by R. Peskin based on the hydrodynamics stability equations, is improved to obtain the optimum condition of ultrasonic atomizing.

2. Solution of Mathieu's Equation

The ultrasonic energy radiated from a vibrator causes a capillary wave on the surface of water when a liquid layer of thickness h is in between an ultrasonic vibrator and a gas layer. The amplitude of the capillary wave is given by[4]

$$\xi = \Xi(t)\sinh[kh]e^{ikx} \tag{1}$$

Here k is wave number and function Ξ is given as a solution of Mathieu's equation.

$$\frac{d^2\Xi(\tau)}{d\tau^2} + [\alpha - 2\beta\cos(2\tau)]\Xi(\tau) = 0 \tag{2}$$

In Eq. (2), $\tau=\omega_0 t/2$. Where σ, ρ, and a are surface tension, density of the liquid, and displacement amplitude of the vibrator surface, respectively, α and β are as follows;

$$\alpha = \frac{4k^3\sigma}{\omega_0^2\rho}\tanh(kh), \quad \beta = 2ka\tanh(kh). \tag{3}$$

The solution of Eq. (2) could be stable or unstable depending on the values of α and β. The ultrasonic nebulizing can be caused when the amplitude is diverged with the unstable solution. In this case, the solution can be assumed as a function with a periodic term and an exponentially diverged term as shown in the following equation.

$$\Xi(\tau) = e^{\mu\tau}p(\tau) \tag{4}$$

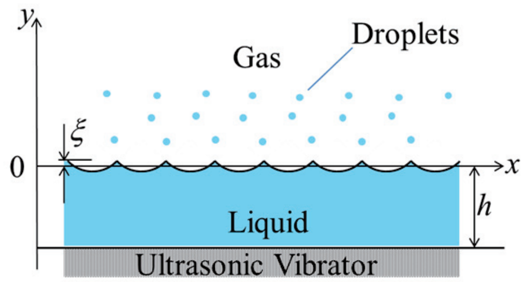

Fig. 1 Theoretical model of ultrasonic nebulizing.

Substituting eq. (4) into eq. (2), the distribution of μ resulted from α and β is calculated numerically as shown in Fig. 2. The dark area in Fig. 2 is corresponding to the stable solutions of Mathieu's equation which is not diverged because $\mu \le 0$. On the contrary, the bright area refers to $\mu > 0$, and corresponds to the unstable solution, which causes ultrasonic atomizing. The brighter the area becomes, the larger the value of μ becomes.

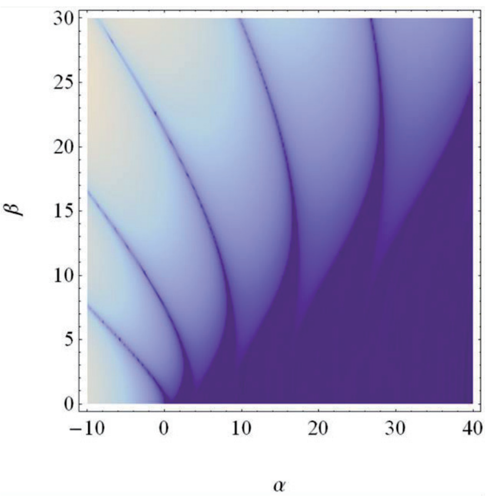

Fig. 2 Distribution of μ resulting from α and β.

3. Droplet size

To obtain the relationship between α and β in Fig. 2, the coordinates (α, β) of the maximum μ in the given β are found and then the coordinates are

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connected as a line. The line is fitted with a square function as a recursion function.

$$\beta^2 = 1.7^2(1-\alpha) \tag{5}$$

Substituting eq. (3) into eq. (5) with approximation of because generally $kh > 3$,

$$\frac{4\sigma}{\omega_0^2\rho}k^3 + \frac{4a^2}{2.89}k^2 - 1 = 0 \tag{6}$$

Because the droplet diameter d and wavelength k are related to $d=\pi/k$, Eq. (6) can be represented as follows.

$$d^3 - c_1 d - c_2 = 0\,. \tag{7}$$

Here, $c_1 = \dfrac{4\pi^2 a^2}{2.89}$, $c_2 = \dfrac{\pi\sigma}{f_0^2\rho}$.

The solution of eq. (7) can be obtained by

$$d = \frac{\left(\frac{2}{3}\right)^{1/3} c_1 + \left(\frac{1}{18}\right)^{1/3}\left(9c_2 + \sqrt{(9c_2)^2 - 12c_1^3}\right)^{2/3}}{\left(9c_2 + \sqrt{(9c_2)^2 - 12c_1^3}\right)^{1/3}} \tag{8}$$

The condition of the diameter for the real value is

$$0.16\frac{\sigma}{\rho} \geq a^3 f_0^2\,. \tag{9}$$

4. Results for water

The calculation result for water with eq. (9) is shown in Fig. 3. In the calculation, surface tension and density of water were 72×10^{-3}N/m and 1000 kg/m^3, respectively. In these results, as the amplitude and frequency of the vibrator are increased, the atomizing occurred vigorously, whereas the ultrasonic atomizing did not occur when the amplitude and frequency were less than a certain level. The droplet size distribution for water is calculated with Eq. (8), and is shown in Fig. 4. In Fig. 4, as the amplitude of vibration displacement increases, the droplet size increases, whereas the size decreases as the driving frequency increases. Those tendencies in the results agree with those of the experimental results in other researches.

5. Summary

To obtain the optimum driving condition for ultrasonic nebulizing, the capillary wave caused by ultrasound on the water surface was analyzed theoretically. The condition, in which the solution of Mathieu's equation diverges with time, was investigated. From that condition, the ranges of the driving frequency and the vibration displacement were obtained numerically, and the droplet size distribution was obtained in these ranges. Applying the result of the investigation, the droplet size distribution of water was calculated. It can be confirmed that the experimentally measured droplet sizes in other studies are within the size distribution calculated in this study. The theoretical analysis in this study could be applied to obtain the optimum driving conditions to make the required droplet size in the ultrasonic nebulizing.

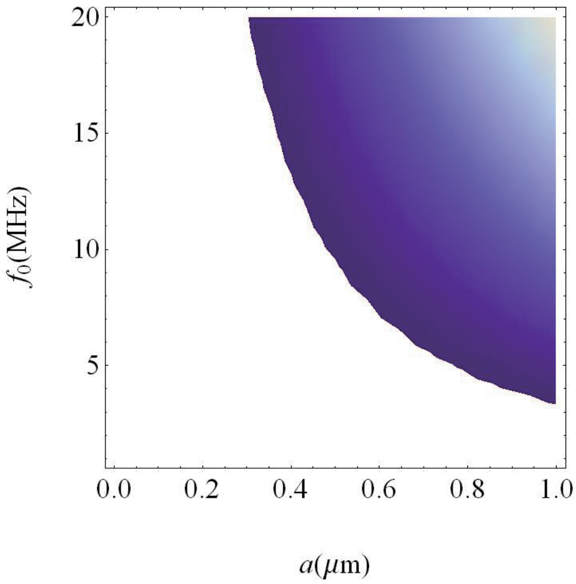

Fig. 3 Ultrasonic atomization condition for water.

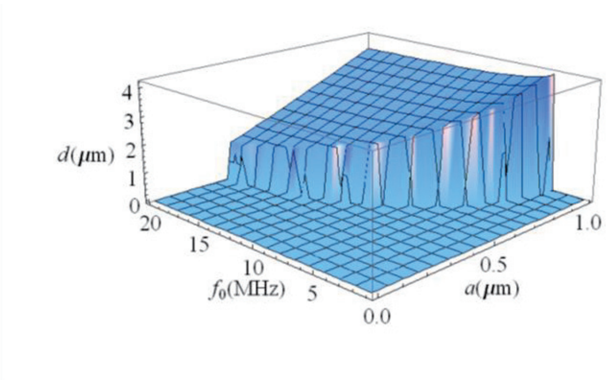

Fig. 4 Droplet size distribution for water.

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