

An Analysis of Ultrasonically Rotating Droplet with Moving Particle Semi-implicit and Distributed Point Source Method

粒子法と分布点音源法による超音波浮揚液滴回転のシミュレーション

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1. Introduction

Ultrasonic levitation has recently been drawing attention as a way of non-contact transportation of small objects, such as liquid droplets, in bioengineering and manufacturing industry. The small objects in the finite amplitude sound field have been known to be trapped near the pressure node of the standing wave with the effect of acoustic radiation force [1-2]. Many experimental reports [3] are presented related to the droplet levitation and their shape and streaming field on the droplet. The droplet with large volume is reported to rotate along its spheroidal axis when they are exposed in the intense sound pressure field. Biswas, et. al [4] experimentally discussed the droplet rotation using an ultrasonic vibrator and a pair of acoustic driver, which works as an levitator and an artificial visco-acoustic generator, respectively. There is so many reports on the acoustic streaming analytical reports near the levitating droplet [5,6], however, most of them has performed on the static grid placed on the steady droplet. Almost no report carried out dynamical streaming simulation on the droplet accompanied by the shape change of streaming domain.

In this paper, the levitated droplet shape and rotational streaming is simulated by coupling two gridless analysis methods, the one is distributed point source method (DPSM, [7]) and the other is moving particle semi-implicit (MPS, [8]) method. The analytical consideration method of the visco-acoustic torque due to the viscous boundary layer is also suggested.

2. Calculation Procedure

Acoustic radiation stress Π and pressure P_{rad} are expressed by the sound pressure p and particle velocity \mathbf{u} , with \mathbf{I} , ρ_0 , c , ω , k , and V_0 are the unit tensor, density, sound speed, angular frequency, wavenumber in air, and vibration velocity of the transducer as

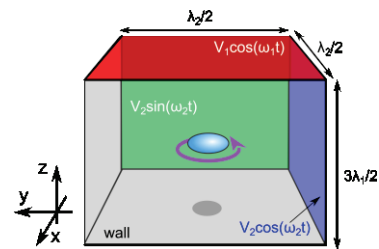


Fig. 1 Problem geometry for the ultrasonic droplet levitation

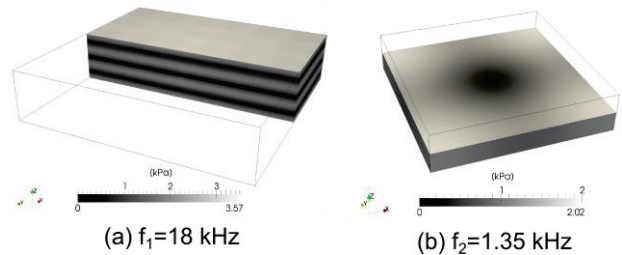


Fig. 2 Sound pressure distribution from (a) ultrasonic levitator and (b) acoustic driver.

$$\Pi = P_{rad} \mathbf{I} + \rho_0 \langle \mathbf{u}^t \mathbf{u} \rangle,$$

$$P_{rad} = \frac{\langle p^2 \rangle}{2\rho_0 c^2} + \rho_0 \frac{\langle \mathbf{u}^2 \rangle}{2}. \quad (1)$$

The sound pressure and particle velocity are calculated using DPSM, where the slight modulation needed to be considered due to viscous boundary layer,

$$p = \sum_i \frac{e^{-jkr}}{r} \{A_i\}, \quad (2)$$

$$u_{\parallel} = \sum_i \frac{(\tilde{\mathbf{e}}_{\parallel} \cdot \tilde{\mathbf{r}})}{j\omega\rho_0} \left(jk + \frac{1}{r} \right) \frac{e^{-jkr}}{r} \{A_i\}, \quad (3)$$

$$u_{\perp} = \sum_i \left[\left(\tilde{\mathbf{n}} + \frac{jk}{\beta^*} \tilde{\mathbf{k}} \right) \cdot \tilde{\mathbf{r}} \right] \left(jk + \frac{1}{r} \right) \frac{e^{-jkr}}{r} \{A_i\}, \quad (4)$$

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$$\mathbf{k} = \mathbf{n} \times \mathbf{r} \times \mathbf{n},$$

$$\beta^* = \frac{\beta_0 + (\rho_0/\rho_w)\beta_w}{1 - (\rho_0/\rho_w)}, \beta = \left[\left(\tilde{\mathbf{k}} \cdot \tilde{\mathbf{r}} \right) k \right]^2 - \frac{\omega}{j\nu}, \quad (5)$$

where A_i , \mathbf{n} , \mathbf{e} are the amplitude of the distributed point sources, surface normal vector, and base vector, respectively. The vector with tilde $\tilde{\mathbf{x}}$ indicates that the vector \mathbf{x} is normalized.

Calculated acoustic radiation pressure is considered in static fluid analysis, which is simulated in MPS, as

$$\frac{DU}{Dt} = -\frac{\nabla P}{\rho_w} + \nu \nabla^2 \mathbf{U} - C\mathbf{U} + \mathbf{g} + \mathbf{f}_R,$$

$$\mathbf{f}_R = \frac{\rho_0}{\rho_w} \frac{\partial \langle \mathbf{u}'\mathbf{u}' \rangle}{\partial \mathbf{n}} \approx \frac{\rho_0}{\rho_w} \frac{\langle \mathbf{u}'\mathbf{u}' \rangle}{(4/3)dr}, \quad (6)$$

$$\nabla^2 P = \frac{\rho_w}{\Delta t} \text{div } \mathbf{U}, \quad P = P_{rad} + \sigma \kappa \quad (\text{on } \Gamma),$$

where P , \mathbf{U} , ρ_w , σ , ν , κ , \mathbf{g} , C and Γ are the static pressure, velocity, density, surface tension, dynamic viscosity of the liquid, curvature of the droplet surface, gravity, air resistance, and set of point on boundary.

3. Results

Fig. 1 indicates the problem geometry for the ultrasonic droplet levitation and rotation. The upper and lower plane on $z = \pm 3\lambda_1/4$ works as an ultrasonic levitator which is driven on the frequency of f_1 . **Fig. 2 (a)** shows the sound field excited by this levitator. The pressure distribution shows third-mode plane standing wave along z -axis and uniform on z -plane.

The plane on $x = \pm \lambda_2/4$ and $y = \pm \lambda_2/4$ in **Fig. 1** works as a pair of acoustic driver with the phase difference of $\pi/2$ between x -planes and y -planes. **Fig. 2 (b)** shows the sound field excited by the driver. Two phase drive create a zero-pressure plane rotating along z -axis, as a result, one nodal point at the coordinate origin is observed in the figure.

$f_1 = 18$ kHz and $f_2 = 1.35$ kHz are chosen to be the same drive condition as Ref. [4]. The amplitude of the acoustic driver is set to 10 times larger than Ref. [4] to observe the effect of visco-acoustic torque in earlier calculation time when the calculation convergence is much satisfied.

Fig. 3 (a) (b) shows the radiation pressure P_{rad} and Reynolds stress force \mathbf{f}_R distribution. Due to negative static pressure around $z=0$ plane in **Fig. 3 (a)**, the droplet is expected to turn into spheroid. \mathbf{f}_R force shows rotation along $-z$ direction, the droplet is expected to rotate along $-z$ direction. **Figs. 4** shows the streaming distribution of the droplet. The shape change and rotation of the droplet is successfully simulated at the same time. The

terminal rotating speed in this case is 0.3 rps.

4. Conclusion

The streaming on an ultrasonic levitated droplet was simulated using MPS and DPSM in three dimensional space. The droplet rotates along the spheroidal axis of the deformed droplet, which agrees in tendencies with the experiment known in Ref. [4].

References

1. L.V. King: Proc. R. Soc. A 147 (1934) 212.
2. W.L. Nyborg: J. Acoust. Soc. Am. 42 (1967) 947.
3. Y. Yamamoto, Y. Abe, A. Fujiwara, K. Hasegawa and K. Aoki: Microgravity Sci. Technol. 20 (2008) 277–280.
4. A. Biswas, E.W. Leung, and E.H. Trinh: J. Acoust. Soc. Am. 90 (1991) 1502.
5. J. Holtmark, I. Johnsen, T. Sikkeland, and S. Skavlem: J. Acoust. Soc. Am. 26 (1954) 26.
6. A. Y. Rednikov and S.S. Sadhal : J. Fluid Mech. 667 (2011) 426.
7. D. Placko and T. Kundu, Eds, DPSM for Modeling Engineering Problems, 1st edition (Wiley-Interscience) (2007).
8. S. Koshizuka, Y. Oka: Nucl. Sci. Eng. 123 (1996) 421.

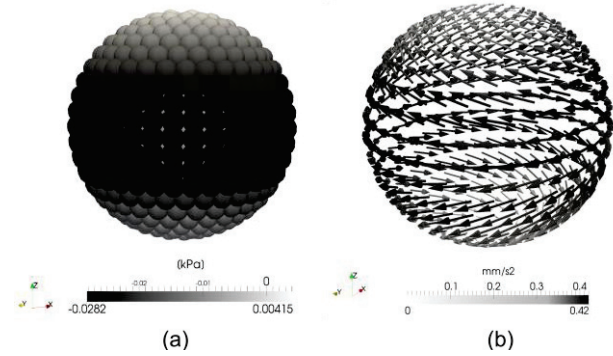


Fig. 3 Initial (a) radiation pressure P_{rad} by ultrasonic levitator and (b) Reynolds force distribution \mathbf{f}_R by acoustic driver.

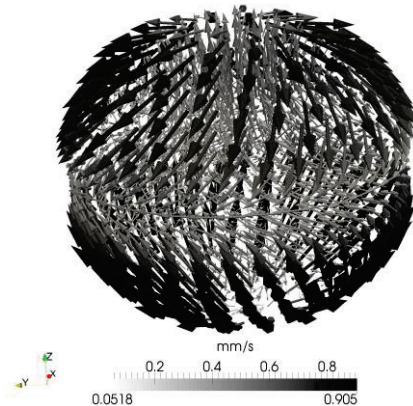


Fig. 4 Streaming distribution on the droplet at time 50 ms.