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Resonant Frequency Analysis of a Lamé Mode Resonator on a Quartz Plate by the Finite-Difference Time-Domain Method with the Collocated Grid Points of Velocities
水晶ラメモード振動子に対する SGCV を用いた FD-TD 法による固有振動解析

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1. Introduction

The finite-difference time-domain (FD-TD) method is a powerful and attractive tool for modeling the propagation and scattering of elastic waves in solids. Discretization of two first-order partial differential equations with finite difference approximations results in a staggered grid (SG)[1], a rotated SG[2], a diagonally staggered grid[3], a Lebedev grid[4], or an SG with collocated grid points of velocities (SGCV)[5-7].

To model elastic wave devices by the FD-TD method, implementation of boundary conditions on planar free surfaces is necessary. For the conventional SG, a rather complicated scheme such as the stress-imaging technique[8] is required, since tangential components of stress are on the free surfaces.

The SGCV, which enables much simpler implementation of boundary conditions, was presented for the FD-TD method[5], and was applied to resonance frequency analysis of a Lamé mode resonator on an isotropic solid to demonstrate the simply imposed boundary conditions on free surfaces[6, 7]. Although many of important materials for elastic wave devices, such as quartz, $LiNbO_3$ or $LiTaO_3$, are anisotropic, the FD-TD method with the SGCV is not applied for elastic waves propagating in anisotropic materials.

In this paper, the FD-TD method with the SGCV is formulated for elastic waves propagating in anisotropic materials. To show the validity of proposed method, resonance frequency of a Lamé mode resonator on a quartz plate is evaluated. For more accurate modeling and efficient computation, rectangular SGCV is also introduced.

2. Numerical Methods

We consider a two-dimensional rectangular Lamé mode resonator on an anisotropic plate as shown in Fig. 1. The Hook's low and the Newton's equations of motion can be written as follows:

$$\Delta_t \frac{\partial T_{x_i x_j}}{\partial t} = R \sum_{k=1}^{2} \sum_{l=1}^{2} C_{x_i x_j x_k x_l} \Delta_{x_l} \frac{\partial v_{x_k}}{\partial x_l},$$

and

$$R \sum_{i=1}^{2} \Delta_{x_i} \frac{\partial T_{x_k x_i}}{\partial x_i} = \Delta_t \frac{\partial v_{x_k}}{\partial t},$$

respectively, where x_i $(i = 1,2)$ denotes the x- and y-coordinates, respectively. Here, $T_{x_i x_j} = \tilde{T}_{x_i x_j}/C_N$, $C_{x_i x_j x_k x_l} = \tilde{C}_{x_i x_j x_k x_l}/C_N$, and $v_{x_k} = \tilde{v}_{x_k}/V_N$ are, respectively, the normalized components of the stress and stiffness tensors and the particle velocity vector, where the variables with a tilde denote the corresponding original variables, $C_N = \max\left\{\tilde{C}_{x_i x_j x_k x_l} | i,j,k,l = 1,2\right\}$, and $V_N = \sqrt{C_N/\rho}$ with the mass density ρ. The Courant number is given as $R = V_N \Delta_t / \Delta$, where Δ_t and Δ_{x_i} are the time and spatial intervals, and $\Delta = \min\{\Delta_{x_i} | i = 1,2\}$.

For the FD-TD analysis, the resonator is discretized with rectangular SGCVs shown in Fig. 2. To discretize the Hook's low and Newton's equations of motion, values of the particle velocity components on the vertices of the cell are interpolated as shown in Ref. 6. To evaluate the particle velocity gradient $\partial v_{x_i} / \partial x_j$ $(i,j = 1,2)$ on grids just inside the free-surface boundaries at the edge of the resonator, the bilinear interpolation with four adjoining grid[6] is used.

3. Numerical Results

We consider quartz as the anisotropic plate and use the following Bechmann's values[9] (in units of $10^9 \mathrm{Nm}^{-2}$) $C_{11}^E = 86.74$, $C_{12}^E = 6.99$, $C_{13}^E = 11.91$, $C_{14}^E = -17.91$, $C_{33}^E = 107.2$,

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$C_{44}^E = 57.94$, and $C_{66}^E = 39.88$. We found that Lamé mode can exist when the Euler's angle is $(0°, -29.347°, 0°)$ and $mb/(na) = 0.9763$ [10], where m and n are natural numbers. Then, the resonant frequency is given as $f_m = [m/(4a)](c/\rho)^{1/2}$ [10]. Here $c = (C_{11}C_{22} - C_{12}^2)/(C_{22} - C_{12})$, where C_{ij} $(i, j = 1,2)$ denotes the stiffness in the above orientation.

We consider the fundamental [m=n=1] Lamé mode. The sizes of the spatial intervals are taken as $\Delta_x = a/50$ and $\Delta_y = b/50$. The x-component of the particle velocity, v_x, is vibrated as a sine-modulated Gaussian pulse given as $\sin(2\pi f_1 t)\exp\left[-(t - t_0)^2/\left(\sqrt{2}w_0\right)^2\right]$ at a point $(-x_0, -y_0)$, where t_0 and w_0 are the center and width of the pulse, respectively. Here, $x_0 = (\lfloor a/(4\Delta_x)\rfloor + 0.5)\Delta_x$ and $y_0 = \left(\lfloor b/(4\Delta_y)\rfloor + 0.5\right)\Delta_y$. In our calculation, R=0.5, $t_0 = 100000\Delta_t$, $\sqrt{2}w_0 = 50000\Delta_t$, and the total number of time step is 400000. The observed time response of v_x at (x_0, y_0) is shown in Fig. 3. After an FD-TD calculation, the frequency spectrum is extracted by the FFT. The spectrum as a function of normalized frequency f/f_1 is shown in Fig. 4. The extracted normalized resonant frequency is 1.0056. We can see the good agreement between the calculated and theoretical values. Fig. 5 shows the distribution of v_x at the final time step of the FD-TD calculation. We can see that the fundamental Lamé mode is excited.

4. Conclusions

The FD-TD method with the SGCV was expanded for anisotropic materials. The proposed method was applied to resonant frequency analysis for a Lamé mode resonator on a quartz plate. The resonant frequency of the fundamental Lamé mode agreed very well with the theoretical value. Consideration of piezoelectricity in our method will be our future work.

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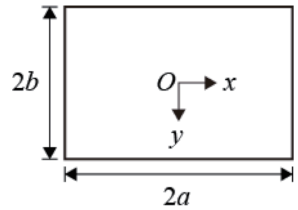

Fig 1 A rectangular Lamé mode resonator.

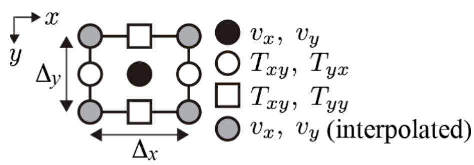

Fig 2 The unit cell of a rectangular SGCV.

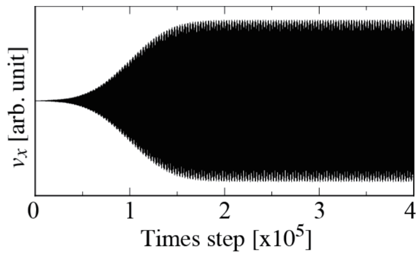

Fig. 3 Time response of v_x as a function of time step.

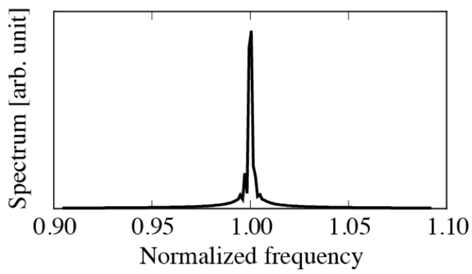

Fig. 4 Extracted spectrum as a function of normalized frequency.

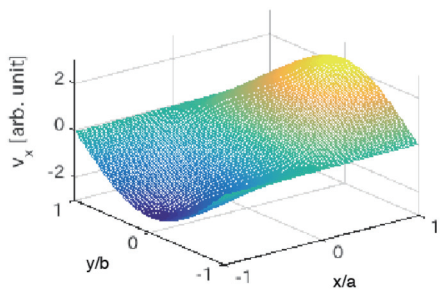

Fig. 5 Distribution of a particle velocity component v_x at the final time step of the FD-TD calculation.