

Evaluation of correlation of LFM signals coded by M-sequences

M 系列でコード化した LFM 信号の相関特性の評価

Kota Yamanaka[‡], Shinnosuke Hirata, and Hiroyuki Hachiya (Tokyo Tech)
山中 航太[‡], 平田 慎之介, 蜂屋 弘之 (東工大)

1. Introduction

In ultrasonic pulse-echo method, pulse compression is employed for improvement of the resolution of the time of flight (TOF) of an echo and the signal-to-noise ratio (SNR) of the echo. Signals which have sharp autocorrelation properties are transmitted and the received signal is correlated with the transmitted signal. In this study, we have studied pulse compression using the signal coded by a maximum length sequence (M-sequence). An M-sequence is one of the pseudo random sequences and generated by a linear feedback shift register (LFSR). Multi-channel measurement is enabled by using ultrasound waves modulated by different M-sequences generated from different LFSRs.¹⁾ However, a number of patterns of LFSRs is small especially in cases of lower-order M-sequences. Therefore, we have proposed multi-channel measurement using linear-frequency-modulated (LFM) signals coded by an M-sequence. In this report, correlation of LFM signals coded by an M-sequence are evaluated.

2. LFM signal coded by M-sequence

An M-sequence is a binary code composed of such as “1” and “-1”. n th-order M-sequence is generated from n -bit LFSR, and its length is 2^n-1 . The transmitted signal is typically coded by modulating amplitude or phase of a sine wave in this code. We can generate uncorrelated transmitted signals by using different M-sequences. Therefore, the number of channels of concurrent measurement is the number of patterns of LFSRs; enough channels cannot be generated especially in lower-order M-sequences, as shown in **Table 1**.

Then, generation of transmitted signals by using not a sine wave but an LFM signal has been proposed. The LFM signal is shown as

$$C_{lfm}(t) = \sin\left\{2\pi\left(f_1t + \frac{f_2 - f_1}{2l_0}t^2\right)\right\}.$$

The frequency changes from f_1 to f_2 , where l_0 is the length of LFM signal. In the case of the coded LFM signal, 1 signal period is assigned as “1” or “-1” of the M-sequence, as shown in **Fig. 1**. Therefore, transmitted signals are uncorrelated when the same M-sequences which are circular-shifted are used. Then, the number of channels of concurrent measurement is the product of the number of LFSR and length of M-sequence.

Table 1 Patterns of LFSR to generate M-sequence.

Order	3	4	5	6	7	8	9	10
LFSR	2	2	6	6	18	16	48	60

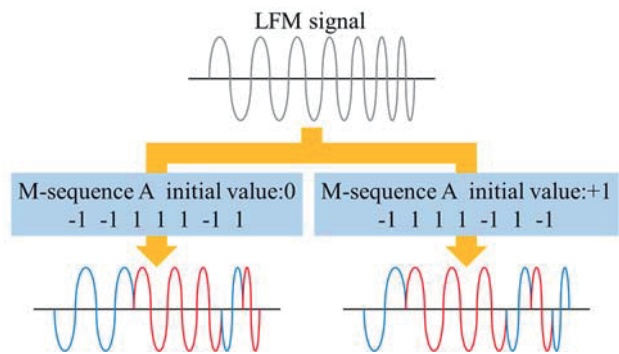


Fig. 1 LFM signals coded by the M-sequence.

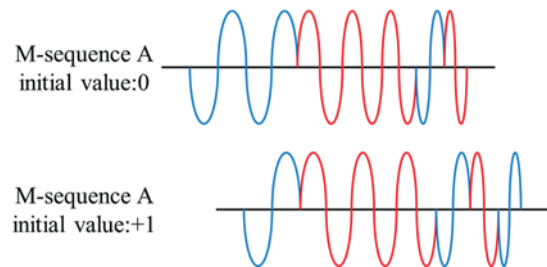


Fig. 2 Diagram of LFM signals coded by the M-sequence whose difference of number of circular shift is 1.

In case the number of the circular shift is small as shown in **Fig. 2**, however, correlation value between transmitted signals might become higher.

3. Correlation property of LFM signals coded by M-sequences

Firstly, in order to evaluate the cross-correlation property of LFM signals coded by the M-sequence, we consider the autocorrelation property of LFM signal. When l_0 is enough long, an autocorrelation function of LFM signal is shown as

$$AC_{lfm}(t) = \frac{\sin(2\pi f_2 t) - \sin(2\pi f_1 t)}{2\pi(f_2 - f_1)t}.$$

Figure 3 shows autocorrelation functions whose center frequency is 50 kHz and sweep band widths Δf are

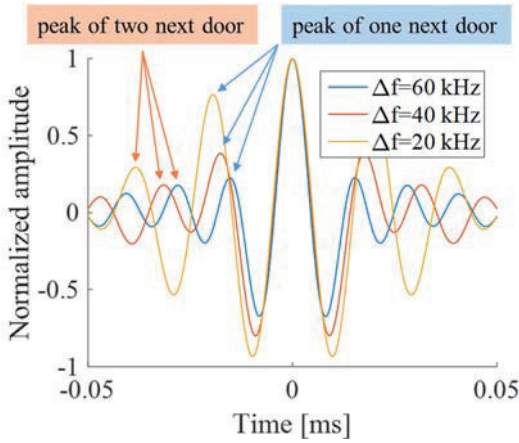
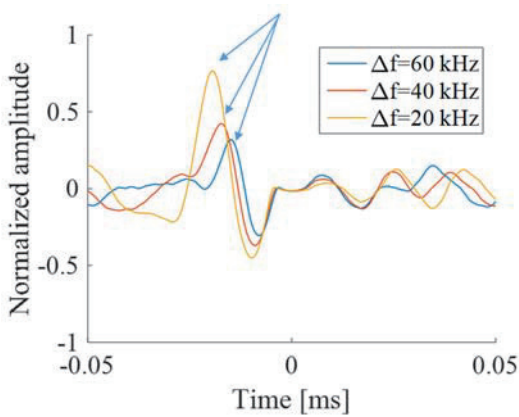
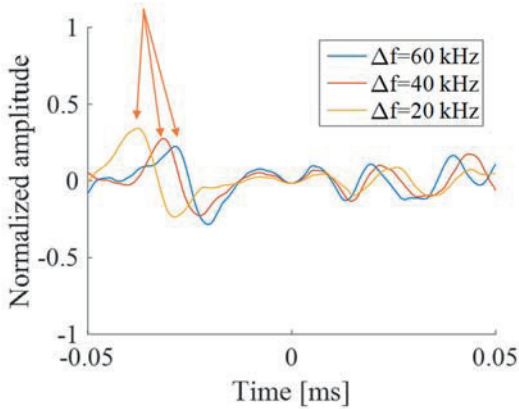


Fig. 3 Autocorrelation function of LFM signal.



(a) difference number of circular shift is 1.

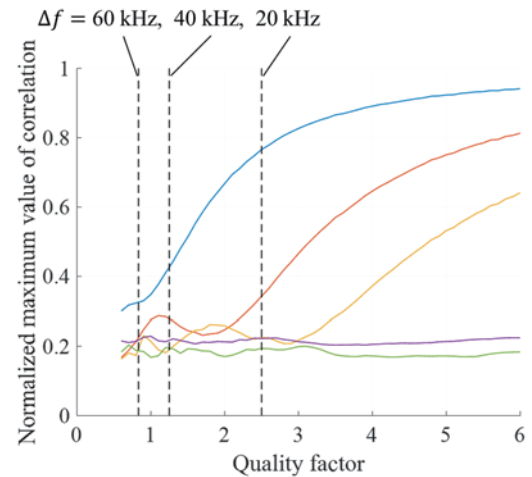


(b) difference number of circular shift is 2.

Fig. 4 Cross-correlation functions of coded LFM signals.

from 20 to 60 kHz. The autocorrelation property of LFM signal depends on the central frequency and the sweep band width.

Secondly, we consider the cross-correlation property of LFM signal coded by the M-sequence. **Figure 4 (a), (b)** shows cross-correlation functions of LFM signals coded by M-sequences whose circular shifts are difference are 1 and 2. Correlation values of one next door from autocorrelation peaks indicated by



(a) cross-correlation
difference number of circular shift is 1 2 3
different LFSR
(b) autocorrelation without correlation peak

Fig. 5 Maximum value of cross-correlation and autocorrelation of LFM signal coded by M-sequence.

blue arrows in **Fig. 3** correspond to cross-correlation peaks whose difference number of circular shift is 1. Furthermore, correlation values of two next door from autocorrelation peaks indicated by red arrows in **Fig. 3** correspond to cross-correlation peaks whose difference number of circular shift is 2. Therefore, in the case of small sweep band width, correlation value which is away from autocorrelation peak for the difference number of circular shift impinges on cross-correlation function.

Figure 5 (a) shows relation between quality factor (a value obtained by dividing center frequency of LFM signal by sweep band width) and cross-correlation value of LFM signals coded by the M-sequence whose difference number of circular shift is small, and those coded by the different LFSR. Maximum values of cross-correlations are low at small quality factor and large difference number of circular shift. **Figure 5 (b)** shows relation between quality factor and autocorrelation value of LFM signal coded by M-sequence without correlation peak. A maximum value of autocorrelation is low during any quality factor.

4. Conclusion

In this report, we evaluated the correlation property of LFM signals coded by the M-sequence. Correlation values depend on quality factors and difference numbers of circular shift.

References

1. T. Hayashi, *et al.*: Proc. of Symposium on Ultrasonic Electronics, **33** (2012), pp. 335-336.