3P4-15

Proceedings of Symposium on Ultrasonic Electronics, Vol. 36 (2015)
5-7 November, 2015

Piezoelectric nonlinear vibration focusing on the second harmonic vibration mode

2 次高調波振動モードに着目した圧電非線形振動の研究

Ryohei Ozaki[1†], Yaoyang Liu[1] and Takeshi Morita[1]
([1] Graduate School of Frontier Sciences, The Univ. of Tokyo)
尾崎　亮平 [1†], 劉　耀陽 [1], 森田　剛 [1] ([1] 東京大学大学院　新領域創成科学研究科)

1. Introduction

The piezoelectric materials are used in resonant type devices which are driven under high power condition. Usually, these piezoelectric devices are designed using FEM. In FEM calculation, only the linear piezoelectric effect is taken into consideration. However when the piezoelectric devices are driven under high power condition, the nonlinear piezoelectric vibrarion becomes appearent. In that case, the simulated results differ from the actual one. To overcome this problem, the FEM involving nonlinear piezoelectric vibration should be developed. The purpose of this study is to establish the nonlinear model of the piezoelectric viblation for FEM. Differently from the privious studies focusing on the third term nonliniarity[1)2)], we studied about the second term nonlinearity because it is larger than that of third term coefficient.

2. Nonlinear model

In this study, the plate type piezoelectric transducer was utilized (Hard type, C203 Fuji ceramics). The material constants of this transducer are shown in **Table 1**. In this model, x origin is the center of the transducer and we measured the longitudinal vibration. The heat generation also affects the nonlinearity[3)]; therefore to eliminate the effect of heat generation, we established the measurement system using function generator (WF1948), laser doppler velocimeter (NLV2500), high speed amplifier (HSA4052) and lock-in amplifier (LI5640). In this system, the interval time between vibration excitation can be controlled. With this control, the temperature was kept constant.

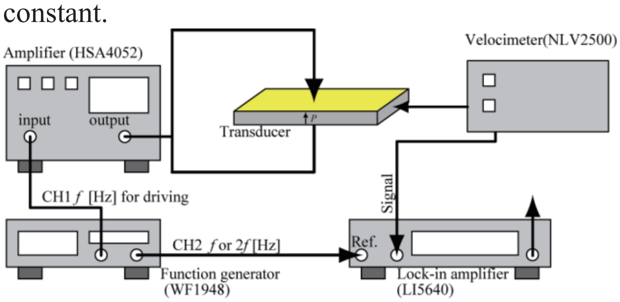

Fig. 1 Measurement

Table1 Material constant

Stiffness c_{11}^E	Density ρ	Q_m
7.9×10^{10} N/m^2	7.7×10^3kg/m^3	2000

In the linear model, the stress and the strain are expressed as proportional,

$$S_1\left(=\frac{\partial u}{\partial x}\right)=s_{11}^E T_1\,. \quad (1)$$

where S_1,T_1, s_1 and u are the strain, the stress, the compliance and the vibration displacement ,respectively.

We added the second term ($s_{11}^{E(2)}T^2$) to explain the effect of the second harmonic vibration mode as

$$S_1=s_{11}^E T_1+s_{11}^{E(2)}T_1^2. \quad (2)$$

By integrating this equation, we obtain the vibration displacements as a function of x,

$$u(x)=\int_0^x s_{11}^E T_1\,dx+\int_0^x s_{11}^{E(2)}T_1^2dx. \quad (3)$$

In this equation, the part of second harmonic (u_2) is defined as

$$u_2(x)=\int_0^x s_{11}^{E(2)}T_1^2dx. \quad (4)$$

The stress T_1 was adopted based on the liner model.

$$T_1(x)=T_{a1}\cos\left(\frac{\omega_r}{c}x\right)\cos(\omega t) \quad (5)$$

where ω_r is resonant frequency and c is sound velocity, respectively.

Finally the second harmonic frequency on each position x is deduced as

$$u_2(x,t)=s_{11}^{E(2)}T_{a1}^2\left(\frac{x}{2}+\frac{L}{4\pi}\sin\left(\frac{2\pi x}{L}\right)\right)\cos(2\omega t) \quad (6)$$

where L is transducer length.

By measuring the displacement of second harmonic vibration, we can calculate the nonlinear parameter.

3. Mode shape of the second harmonic vibration

To obtain the nonlinear parameter, we measured the mode shape of second harmonic vibration at the resonant frequency (35.5kHz) with 100Vpp input voltage. This transducer is symmetric so we measured the half of the transducer. **Figure. 2** shows the mode shape of the transducer and the fitting curve with our model.

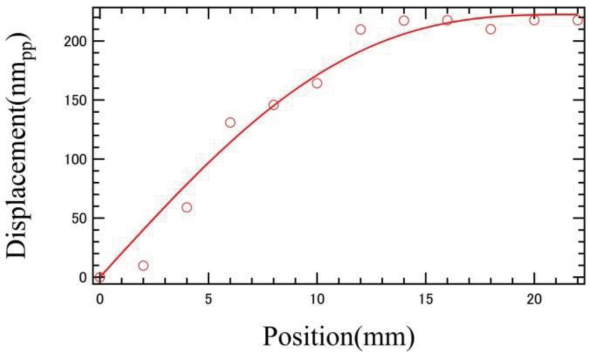

Fig. 2 Mode shape of the second harmonic vibration

The fitting result was well fitted and the value was calculated to be 3.6×10^{-21} [m^4/N^2] from this measurement.

4. The $s_{11}^{E(2)}$ calculation from the velocity at the tip of the transducer

The amplitude of the stress at the center of transducer (T_{a1}) and that of velocity with driving frequency at the tip (v_{a1}) are also proportional in linear model expressed as

$$T_{a1} = \rho c v_{a1}. \tag{7}$$

From the equation (6), the displacement of the second harmonic vibration at the tip of transducer is

$$u_2(\frac{L}{2}, t) = s_{11}^{E(2)} T_{a1}^2 \frac{L}{4} \cos(2\omega t). \tag{8}$$

By calculating the temporal differentiation of equation (8), the velocity is expressed as

$$v_2\left(\frac{L}{2}, t\right)\left(= \frac{\partial u_2(L,t)}{\partial t}\right) = s_{11}^{E(2)} T_{a1}^2 \frac{L}{2} \omega \sin(2\omega t)$$
$$= v_{a2} \sin(2\omega t) \tag{9}$$

From equation (7) and (9), the relationship between v_{a2} and v_{a1} is expressed as

$$\frac{v_{a2}}{\omega} = s_{11}^{E(2)} \frac{\rho^2 c^2 L}{2} v_{a1}^2 \tag{10}$$

Using a lock-in amplifier (LI5640), we measured the velocity with the driving frequency and that with the second harmonic frequency. The measurement result and the fitting curve with equation (10) were shown in **Fig. 3**. From this curve fitting ,the calculated value of $s_{11}^{E(2)}$was calculated to be 3.4×10^{-21} [m^4/N^2]. This value was the almost same to the value calculated from mode shape measurement (3.6×10^{-21} [m^4/N^2]). This curve fitting indicates that the value of $s_{11}^{E(2)}$ can be treated as a constant parameter regardless of frequency.

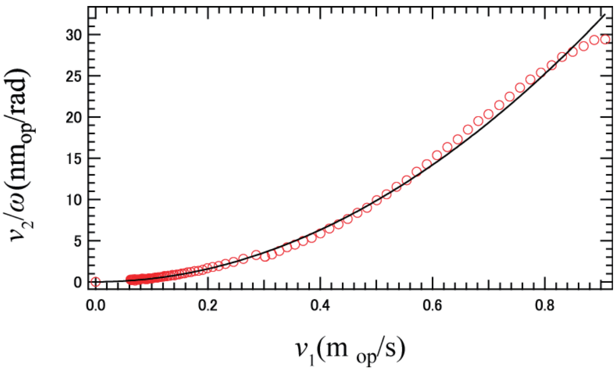

Fig3. Relationship between v_{a2} and v_{a1}

5. Conclusion

In this study, we established the second harmonic vibration model. This model was verified by calculating $s_{11}^{E(2)}$ from the mode shape measurement and the velocity at the tip of the transducer. During the measurements, the temperature was kept constant to eliminate the effect of temperature increase. To introduce this nonlinear model to the FEM, the effect of the heat generation while driving should be considered in the next study.

References

1. Keisuke Ishii, Norihito Akimoto, Shinjiro Tashirio and Hideji Igarashi: "Jump Phenomena of Current Piezoelectric Ceramic Vibrators Under High Power Conditions", *Journal of the European Ceramic Society*, vol 19 pp1157-1160 (1998)
2. Yaoyang Liu and Takeshi MORITA, "Nonlinear coefficients in lead-free CuO-(K,Na)NbO$_3$ transducers", *Jpn. J. Appl. Phys,* vol54 07HC01 (2015)
3. D.A.HALL: "Review Nonlinearity in piezoelectric ceramics" *Proceedings of the 2001 Journal of Materials Science* vol*36*, pp. 4575-4601, 2001