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Evaluation model of the high power characteristics considering the interaction between temperature rise and nonlinear vibration

温度上昇と非線形振動の相互影響を考慮した圧電体のハイパワー特性評価モデル

Susumu Miyake† and Takeshi Morita
(Graduate School of Frontier Sciences, The Univ. of Tokyo)
三宅 奏†，森田 剛 (東京大学大学院　新領域創成科学研究科)

1. Introduction

Piezoelectric transducers are used in various devices and they are driven under high power condition. Nonlinear effects such as "jumping phenomena" in current, hysteresis of admittance curve and temperature rise of piezoelectric transducer under high power driving are well-known; however, they cannot be simulated by a traditional piezoelectric equivalent circuit. Our research group has already succeeded in simulating nonlinear effects neglecting temperature rise[1]. However, this method is limited for uniform temperature condition. In the actual driving, temperature distribution is inevitable in the transducer.

In this study, we established a high power piezoelectric vibration model considering the interaction between temperature rise and nonlinear effect. The plate-type transducer was driven by longitudinal 31 mode. First, temperature-dependent material constants of PZT transducer was measured. Second, we formulated the nonlinear vibration using a distributed parameter system with a transfer matrix method. Using the transfer matrix model and considering the change of material constants due to temperature increase, we succeeded in simulating the piezoelectric vibration under high power driving.

2. Nonlinearity

The relationship between applied voltage V and current i in the mechanical port of the piezoelectric LCR equivalent circuit is given as equation (1):

$$L\frac{di}{dt}+R_0 i+\frac{1}{C_0}\int idt=V \tag{1}$$

where L, R_0 and C_0 are equivalent mass, equivalent dumping and equivalent elastic constant, respectively. Under high power condition, the relationship is written as equation (2)[1]:

$$L\frac{di}{dt}+R_0 i+\eta i^3+\frac{1}{C_0}\int idt+\xi\omega^3\left(\int i\,dt\right)^3=V \tag{2}$$

where ω is driving angular frequency; ξ and η are nonlinear coefficients which are related to the mechanical third mode vibration excited by high stress. Nonlinear coefficients, ξ and η are related to mechanical stiffness and loss, respectively and they are considered under lumped parameter system[1]. In this research, we considered the temperature distribution of the piezoelectric transducer with distributed parameter system. Accordingly, we added the nonlinear coefficient E_3into the equation between stress T_1 and strain $\frac{\partial u}{\partial x}$ as equation (3):

$$T_1=E_1\frac{\partial u_x}{\partial x}+E_3\left(\frac{\partial u_x}{\partial x}\right)^3 \tag{3}$$

where x is longitudinal position and u_x is displacement. E_1 and E_3 are complex number. Real and imaginary part of E_3 is given as equation (4)-(5):

$$E_{3r}=\frac{16}{9}\frac{E_{1r}^2A^2}{\rho\omega L}\xi \tag{4}$$

$$E_{3i}=\frac{16}{9}\frac{E_{1r}^2A^2}{\rho\omega L}\eta \tag{5}$$

where E_{1r} is real part of E_1, A is force factor and ρ is density.

3. Measurement

Material constants of the piezoelectric transducer can be obtained from admittance curve measurement. We measured the admittance curve of the PZT transducer under various temperature condition and obtained temperature dependency of material constants. We put a rectangular plate type PZT transducer (Fuji Ceramics PZT C-203 44mm × 7mm × 2mm) in the thermostat chamber (Yamato DKN302) and connected it to the frequency response analyzer (NF FRA5097). We measured the admittance curve with linear effect condition (input voltage: 0.5 [V_{pp}]) and high nonlinear effect condition (input voltage: 10 [V_{pp}]). They were measured in a short time to avoid temperature rise during the measurement. **Figure 1** shows the

miyake@ems.k.u-tokyo.ac.jp

temperature dependency of nonlinear coefficients ξ and η. We utilized these results for the calculation to consider the temperature dependency.

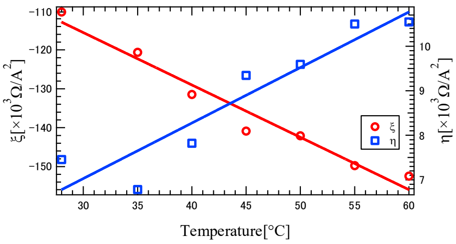

Fig. 1. Temperature dependency of nonlinear coefficients

4. Transfer matrix model for nonlinear vibration

We used the transfer matrix to calculate the piezoelectric nonlinear vibration as a distributed parameter system. We defined Young's modulus including a nonlinear term as equation (6) :

$$E'(x) = E_1 + \frac{3}{4}E_3\left(\frac{\partial u_x(x)}{\partial x}\right)^2 \tag{6}$$

It can be obtained from equation (3) by eliminating the higher frequency component. Using this Young's modulus E', the transfer matrix including nonlinear component is given as equation (7) :

$$\begin{pmatrix} F_1 \\ v_1 \\ V_0 \\ I_1 \end{pmatrix} = \begin{pmatrix} a & -\frac{SZ'}{j}b & (1-a)A & 0 \\ -\frac{b}{jSZ'} & a & \frac{b}{jSZ'}A & 0 \\ 0 & 0 & 1 & 0 \\ \frac{b}{jSZ'}A & (1-a)A & j\omega C_d - \frac{b}{jSZ'}A^2 & 1 \end{pmatrix} \begin{pmatrix} F_2 \\ v_2 \\ V_0 \\ I_2 \end{pmatrix} \tag{7}$$

where $a = \cos(k'l)$, $b = \sin(k'l)$ and sound velocity c', wave number k', force factor A, acoustic impedance Z' and cross-sectional area S are given as:

$$c' = \sqrt{\frac{E'}{\rho}},\ k' = \frac{\omega}{c'},\ A = \frac{wd_{31}}{s_{11}^E},\ Z' = \frac{E'}{c'},\ S = wh \tag{8}$$

In this calculation, the transducer was divided into 101 parts and material constants were dependent on temperature at each divided part. Calculating velocity and force at each surface, the stress and the strain distribution could be obtained.

5. Calculation

Changing material constants by temperature distribution and calculating the vibration state with transfer matrix model, we can handle the nonlinear vibration with temperature dependency. The temperature distribution was calculated by heat conduction equation (9):

$$\rho\gamma\frac{\partial T}{\partial t} = \lambda\frac{\partial^2 T}{\partial x^2} + \dot{q} - \alpha S_{hc}(T - T_\infty) \tag{9}$$

where γ is specific heat capacity, $T(x,t)$ is tempeature, λ is heat conductivity, α is convection heat transfer coefficient, S_{hc} is heat conduction area and T_∞ is atmospheric temperature. **Figure 2** shows the calculated and measured temperature distribution. At the center of the transducer, temperature was calculated 54.4℃ and measured value was 59.3℃. Using this method, we could simulate the saturation of the vibration velocity with and without temperature rise as shown in **Fig. 3**. As the input voltage increases, temperature increase due to the vibration loss. Consequently, the interaction between temperature rise and nonlinear effect causes the velocity saturation in lower voltage than the calculation without temperature rise.

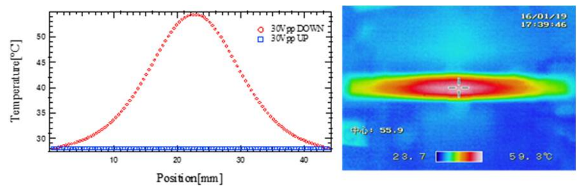

Fig. 2. Calculated and measured temperature distribution (Applied driving voltage: 30Vpp)

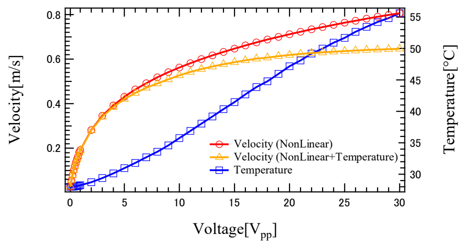

Fig. 3. Calculated velocity saturation with/without temperature rise at the center of the transducer

7. Conclusion

In this study, we proposed a high power piezoelectric vibration model considering the interaction between temperature rise and nonlinear vibration. It enables to estimate the piezoelectric material suitable for the high power piezoelectric devices, overwhelming the conventional PZT.

References

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