

## Estimation of Sound Velocity Distribution from Time Delay Received at Each Element of Ultrasonic Probe

超音波プローブの素子受信時刻差を用いた音速分布推定

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### 1. Introduction

Ultrasound imaging has high performance in depicting soft tissue, and it can be repeated because of its safety. Therefore, it is very useful in diagnosis of cardiovascular disease and observation of a variety of organs [1]. However, it is known that the image quality in ultrasound imaging deteriorates due to the heterogeneity of sound velocity of tissue *in vivo* [2]. In order to solve this problem, it is necessary to estimate the accurate sound velocity distribution in the imaging area. In this study, we propose a method to estimate of sound velocity distribution from the time delay received at each element of ultrasonic probe.

### 2. Materials and Methods

#### 2.1 Estimation of sound velocity using time delay

In the present study, we use a linear array probe that transmits an ultrasound beam to the direction  $\theta_R$  to the probe surface, as shown in Fig. 1. The proposed method estimates the sound velocity distribution from the delay time of the signal received at each element of ultrasonic probe. First, it determines the arrival times of the echo returned from each strong target using the RF signal received at the central element. Second, it calculates the delay time of each element using the cross correlation function.

The delay time of the  $k$ -th element is expressed by

$$T(x_k) = \frac{1}{c} \left\{ r + \sqrt{x_k^2 - 2r \sin \theta_R \cdot x_k + r^2} \right\},$$

$$(-K \cdot \Delta x \leq x_k \leq K \cdot \Delta x) \quad (1)$$

where  $x_k = \Delta x \cdot k$  ( $\Delta x$  is element pitch) is the lateral position of the  $k$ -th element,  $K$  is the number of the receiving elements on one side of the probe ( $k = -K, \dots, K$ ),  $c$  is the average sound velocity on the propagation path,  $r$  is the distance from the central element to the strong target, and  $\theta_R$  is the direction angle of the target for the central element. Since the forward path from the central element ( $x_k = 0$ ) is the equal to its backward path to the central element, delay time of  $k$ -th element from

that of the central element is expressed by

$$T_R(x_k) = T(x_k) - \frac{T(0)}{2}$$

$$= \frac{1}{c} \sqrt{x_k^2 - 2r \sin \theta_R \cdot x_k + r^2}. \quad (2)$$

The square of Eq. (2) becomes a quadratic function regarding  $x_k$  as follows:

$$f(x_k) = \{T_R(x_k)\}^2 = \frac{1}{c^2} x_k^2 - \frac{2r \sin \theta_R}{c^2} x_k + \frac{r^2}{c^2}$$

$$\equiv a_2 x_k^2 + a_1 x_k + a_0. \quad (3)$$

The coefficients  $a_2, a_1$  and  $a_0$  are determined by fitting the measured  $\{T_R(x_k)\}^2$  to the quadratic function using the least square method. The following acoustic parameters are estimated by

$$\hat{c} = \sqrt{\frac{1}{a_2}}, \hat{r} = \sqrt{\frac{a_0}{a_2}}, \hat{\theta}_R = \sin^{-1} \left( -\frac{a_1}{2\sqrt{a_2 a_0}} \right). \quad (4)$$

This estimation process is applied to each of the selected strong targets.

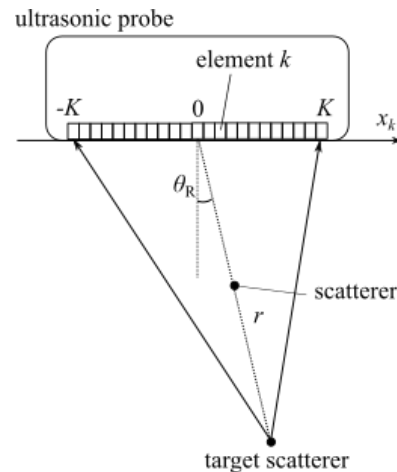


Fig. 1 Schema of the system setup using an ultrasonic probe and target scatterer.

#### 2.2 Sound velocity distribution

Figure 2 shows the measurement grid aligned vertically in the ROI. For each grid, the sound velocity is assumed to be constant and is estimated as follows. When the grid length is  $l$  and the sound velocity in each grid is assumed to be  $c_n$  ( $n = 1, 2, \dots, N$ ), and when the  $m$ -th strong target ( $m = 1, 2, \dots, M$ ) are present in the  $N$ -th grid, the delay time due to one way propagation from the probe

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surface to  $m$ -th target is given by

$$\sum_{n=1}^{N-1} l\tau_n + \{\widehat{r}_m - (N-1)l\}\tau_N = \frac{\widehat{r}_m}{\widehat{c}_m}, (m = 1, 2, \dots, M) \quad (5)$$

where  $\tau_N$  is  $1/c_N$ , and  $M$  is the number of strong targets in the ROI. Eq. (5) is rewritten as a set of simultaneous linear equations as

$$\mathbf{D}\boldsymbol{\tau} = \mathbf{T}, \quad (6)$$

where  $\mathbf{D}$  is a  $M \times N$  coefficient matrix of the path length for the target,  $\boldsymbol{\tau}$  is a  $N \times 1$  vector of  $\{\tau_n\}$  ( $n = 1, 2, \dots, N$ ), and  $\mathbf{T}$  is a  $M \times 1$  vector of  $\{\widehat{r}_m/\widehat{c}_m\}$ . This problem can be solved by the following least square method under the condition of  $N \leq M$ .

The norm of the vector  $\mathbf{D}\boldsymbol{\tau} - \mathbf{T}$ , that is the sum of squares of the residual, is minimized to estimate the sound velocity distribution  $\widehat{\mathbf{c}}$  as follows:

$$\widehat{\mathbf{c}} = \arg \min_{\boldsymbol{\tau}} \alpha, \alpha = |\mathbf{D}\boldsymbol{\tau} - \mathbf{T}|^2. \quad (7)$$

The reciprocal of sound velocity distribution,  $\widehat{\mathbf{c}}$ , is acquired by setting a derivative of  $\boldsymbol{\tau}$  in terms of  $\alpha$  to be zero. Using a singular value decomposition of  $\mathbf{D}$ ,  $\mathbf{D}$  can be expressed as  $\mathbf{D} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$ , where  $\mathbf{U}$  is an orthogonal  $M \times M$  matrix,  $\mathbf{V}$  is an orthogonal  $N \times N$  matrix, and  $\boldsymbol{\Sigma}$  is a diagonal  $M \times N$  matrix. Since the pseudo-inverse matrix of  $\mathbf{D}$  is obtained as  $\mathbf{D}^+ = \mathbf{V}\boldsymbol{\Sigma}^+\mathbf{U}^T$ , where the pseudo-inverse matrix  $\boldsymbol{\Sigma}^+$  of  $\boldsymbol{\Sigma}$ , the reciprocal of the sound velocity distribution  $\widehat{\mathbf{c}}$  is estimated by

$$\widehat{\mathbf{c}} = \mathbf{D}^+\mathbf{T} = \mathbf{V}\boldsymbol{\Sigma}^+\mathbf{U}^T\mathbf{T}. \quad (8)$$

By using  $\widehat{\mathbf{c}}$  and  $c_n = 1/\tau_n$ , the sound velocity distribution of the grids aligned vertically is estimated. The same operation applied to all the adjacent rows constructs a 2D sound velocity distribution.

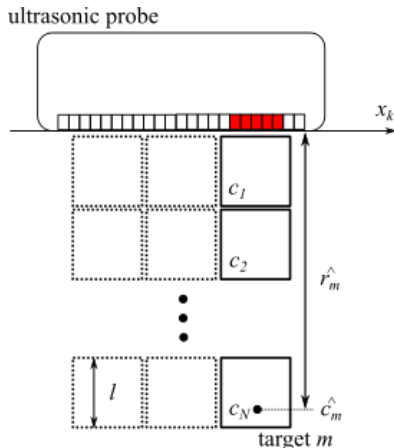


Fig. 2 Measurement grids aligned vertically in a ROI for estimation of the sound velocity distribution.

### 3. Experiment and Result

We employed a phantom (054GS, CIRS) located in water. Nylon wires of a 100- $\mu\text{m}$  diameter are embedded in the phantom. The sound velocity of the phantom is 1,540 m/s, and that of water at 25 °C is 1,498 m/s. A nichrome wire of a 15- $\mu\text{m}$  diameter was set in water as a strong target. We used an ultrasonic equipment (ProSound  $\alpha$  10, Hitachi Aloka) with a linear probe UST-5412, where the center frequency was 10 MHz. 96-ch elements were used for receiving. Two measurement grids were set in the ROI, where the grid size in axial direction was 1.5 cm in length. In this setting,  $M = N = 2$ .

The estimation result is shown in Fig. 3 (b). The sound velocity of the grid in water was estimated to be 1,492 m/s. The sound velocity of the grid in the phantom was estimated to be 1,551 m/s. The estimation errors of sound velocities in water and phantom were respectively 0.4% and 0.7%, showing a high potential of the proposed method in estimating sound velocity distribution.

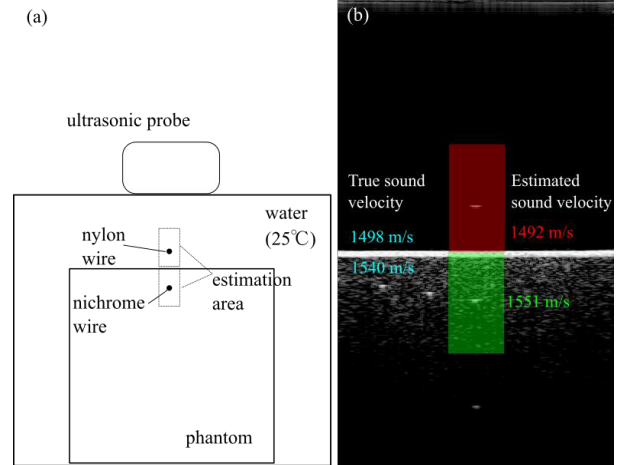


Fig. 3 (a) Experimental system. (b) The result of estimated sound velocities in water and the phantom.

### 4. Conclusion

In this study, we estimated the local sound velocity of two grids from two delay time set received at 96-ch elements of a linear ultrasonic probe. The proposed method succeeded in estimate the sound velocities of water and phantom with estimation error of less than 1%. This result shows a high potential of the proposed method in estimating sound velocity distribution.

### References

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