

# Optimal Design of a Sparse Array to Simulate a Fully Dense Underwater Planar Array Transducer for Underwater Vehicles

Muhammad Shakeel Afzal and Yongrae Roh<sup>†</sup>  
(Kyungpook National University, Korea)

## 1. Introduction

Planar arrays comprising a 2D arrangement of elements have been extensively used in SONAR, radar, and medical applications for the past couple of decades. The radiation pattern of a planar array depends upon its aperture geometry and is greatly influenced by the number, size, and orientation of the array elements. Cost and the complication of the underwater array system are highly dependent on the total number and geometry of the individual elements. In principle, the design of an array with periodic element spacing is simple if the elements are spaced no further than one-half of a wavelength ( $\lambda/2$ ) apart [1].

In this paper, the design and optimization of a 2D sparse planar array of transducers was carried out with a small number of elements to achieve the array performance nearly similar to that of a fully sampled array for underwater acoustical applications. Main performance parameters to evaluate the effective radiation pattern were peak sidelobe level (PSLL) and mainlobe beam width (MLBW). Optimal design of the sparse array was accomplished to satisfy desired performance parameters. The optimal radiation patterns computed analytically were cross-verified by comparing them with those calculated by the finite element method (FEM) for identical operating conditions as in the analytical calculations.

## 2. Sparse Array Optimization

Sparse-array techniques periodically or randomly deactivate some elements of the 2-D array transducer [3]. The resultant radiation pattern can be computed by multiplying the radiation pattern of an array of simple sources with that of an individual element source, i.e. product theorem [4], as specified in Eq. (1).

$$H_{eff}(\theta, \phi) = H_E(\theta, \phi) \times H_T(\theta, \phi) \times H_R(\theta, \phi) \quad (1)$$

where  $H_{eff}$  is an effective array radiation pattern,  $H_E$  is a single element radiation pattern, and  $H_T$  and  $H_R$  are transmitter and receiver array radiation patterns, respectively.

<sup>†</sup>yryong@knu.ac.kr

Optimization of the pulse-echo radiation pattern was carried out using the OptQuest Nonlinear Programming (OQNLP) algorithm with the following optimization parameters [2];

Objective Function:

Minimize the PSLL difference between dense and sparse arrays for three azimuth planes, i.e.  $0^\circ$ ,  $22.5^\circ$  and  $45^\circ$ .

Constraints:

- (a) No. of all active elements = 15
- (b)  $MLBW_{full} - 1^\circ \leq MLBW_{sparse} \leq MLBW_{full} + 1^\circ$

Design variable:

Element Weights,  $(W_{i,j}) = [1 \text{ or } 0]$   
where  $i, j = 1, 2, 3, \dots, 10$

The element is active when the weight is 1 and inactive when the weight is 0. In calculation, three symmetries were incorporated as described in Fig. 1, which reduces the input variables to 15. The Optimized sparse array layout comprised 32 transmitters and 68 receivers. Array symmetries with optimal quarter layout are shown in Fig. 1.

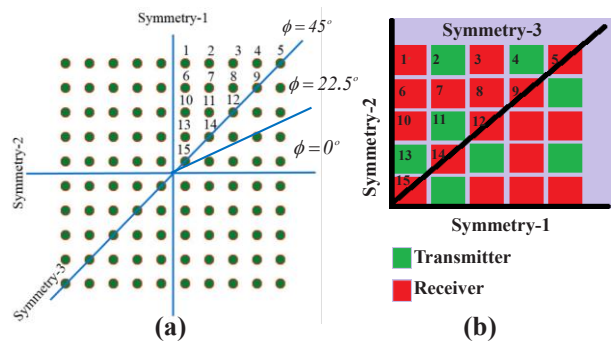


Fig. 1 Planar array layout (a) array symmetries (b) quarter sparse array.

## 3. Finite Element Analysis (FEA) of Array

The planar array quarter model was used for the radiation pattern computation at a far field point. The finite element model of the quarter array is shown in Fig. 2. Radiation patterns were computed using the FEM for three azimuth planes of interest as shown in Fig. 3.

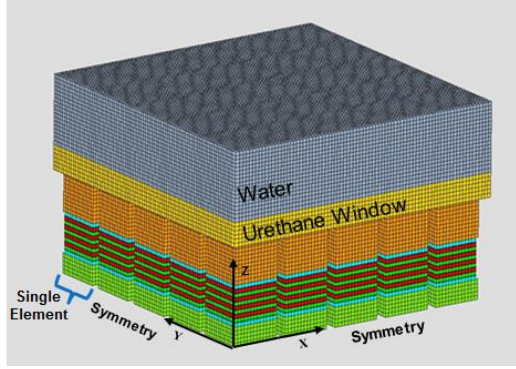


Fig. 2 Finite element model of the quarter planar array.

#### 4. Results and Discussion

Analytical results for dense and optimal sparse arrays were compared with FEA results. Normalized PSLL for dense and sparse arrays were -26.2 dB and -27.9 dB, respectively, from the analytical calculations while -26.9 dB and -28.9 dB, respectively, from the FEA. **Fig. 3** compares the results. The FEA results showed good agreement with analytical results. A small difference between the FEA and analytical results was due to the crosstalk between the array elements, which was not included in the analytical results.

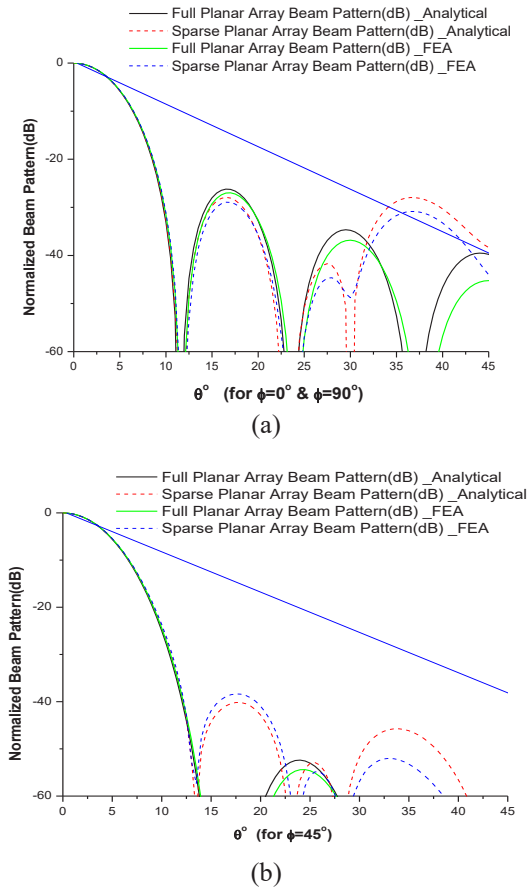


Fig. 3 Radiation pattern comparison for the sparse and dense arrays (analytical vs. FEA)

Similar closeness was achieved for the -6 dB MLBW for analytical as well as FEA results. Sparse array PSLL for 45° was higher than that of the dense array but still much lower than the PSLL at 0° and 22.5° azimuth planes. All the results are summarized in **Table I**.

Table I. Comparison of dense and sparse arrays performance.

Performance Parameter	Array Type (Method)	Azimuth Angle( $\phi$ )		
		0°	22.5°	45°
MSLL (Unit: dB)	Dense (Analytical)	-26.2	-36.8	-52.2
	Dense(FEA)	-26.9	-36.6	-54.4
	Sparse (Analytical)	-27.9	-39.2	-40.2
	Sparse(FEA)	-28.9	-49.3	-38.4
MLBW (Unit: deg)	Dense (Analytical)	11.2	11.2	11.3
	Dense(FEA)	10.1	10.2	10.3
	Sparse (Analytical)	10.2	10.2	10.3
	Sparse(FEA)	10.3	10.3	10.4

#### 4. Conclusions

Optimal design of the sparse array was carried out using the OQNLP algorithm to achieve the performance similar to that of a dense array. Validity of the analytical results was verified through comparison with FEA results.

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