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Effects of curvatures and torsions on dispersion property of guided waves propagating in a helical structure obtained by a semi-analytical finite element method

らせん構造の曲率と捩率がガイド波の分散特性に与える影響の半解析的有限要素解析

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1. Introduction

Guided wave ultrasonic modes propagating over long distances can be applied to non-destructive evaluation(NDE) for wire-ropes. Guided waves have a dispersion property, which represents frequency dependence of propagation velocities. Therefore, for effective use of guided waves, we need to know this property, which is usually expressed as dispersion curves[1]~[2].

Dispersion curves of complex structures including wire-ropes are generally obtained by a semi-analytical finite element (SAFE) method [2] ~[9]. Treyssede et al. discussed dispersion curves in a helical structure obtained by the SAFE method. However, they did not examine effects of curvatures and torsions on the dispersion property of guided waves propagating in a helical structure.The purpose of this study is to numerically investigate such effects.

2. A helical coordinate system

Let (x, y, s) denote a curvilinear coordinate system attached to a curved waveguide, where x and y are the cross-section coordinates and s is the axial coordinate. A helical coordinate system (r, θ, s) can be defined by introducing the helix centerline curve, described by the following position vector in the Cartesian orthonormal basis $(\boldsymbol{e_x}, \boldsymbol{e_y}, \boldsymbol{e_s})$:

$$\boldsymbol{R}(s) = R\cos\left(\theta + \frac{2\pi}{l}s\right)\boldsymbol{e_x} + R\sin\left(\theta + \frac{2\pi}{l}s\right)\boldsymbol{e_y} + \frac{L}{l}\boldsymbol{e_s}, \quad (1)$$

where $l = \sqrt{L^2 + 4\pi^2 R^2}$ is the curvilinear length of one helix step and L is the helical pitch (see Fig. 1). The unit tangent ($\boldsymbol{T}$(s)), normal ($\boldsymbol{N}$(s)), and binormal vectors ($\boldsymbol{B}$(s)) to the centerline are obtained from $\boldsymbol{T}(s) = \frac{d\boldsymbol{R}(s)}{ds}$ and Serret-Frenet formule:

$$\frac{d\boldsymbol{T}(s)}{ds} = -\kappa \boldsymbol{N}(s), \quad (2)$$

$$\frac{d\boldsymbol{N}(s)}{ds} = -\tau \boldsymbol{B}(s) + \kappa \boldsymbol{T}(s), \quad (3)$$

$$\frac{d\boldsymbol{B}(s)}{ds} = -\tau \boldsymbol{N}(s). \quad (4)$$

For helix, the curvature $\kappa = \frac{4\pi^2 R}{l^2}$ and the torsion $\tau = \frac{2\pi L}{l^2}$ are constant.

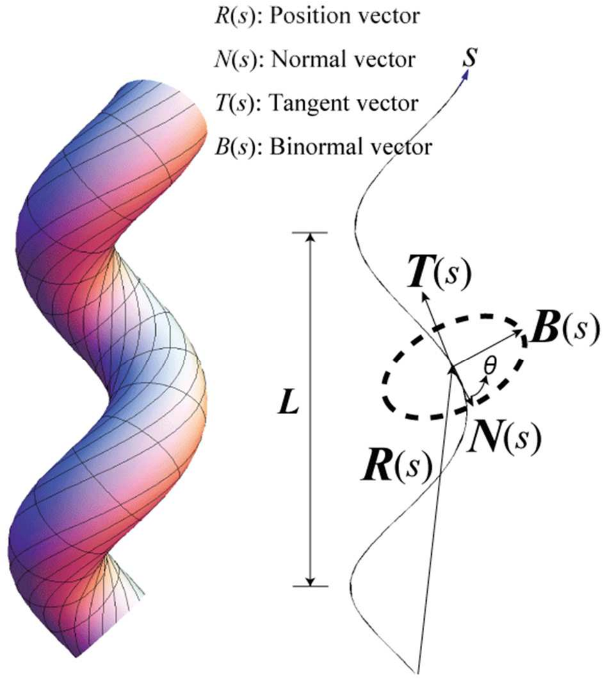

Fig. 1 A helical coordinate

3. A formulation of a semi-analytical finite element method

The SAFE method adopts a harmonic exponential term, $e^{i(ks-\varpi t)}$, to describe properties of guided waves, where s represents the guided wave propagation direction, k represents the wavenumber, ϖ represents the angular frequency, and t represents time.

A governing equation can be obtained through the virtual work principle:

$$\int_V \delta \boldsymbol{u}^T(\rho \ddot{\boldsymbol{u}})dV + \int_V \delta \boldsymbol{\varepsilon}^T \boldsymbol{\sigma} dV = 0. \quad (5)$$

After the FE discretization of the formulation (5), we can develop the expression for a formulation of SAFE:

$$\left(\boldsymbol{K}_1^j + ik\boldsymbol{K}_2^j + k^2\boldsymbol{K}_3^j - \varpi^2\boldsymbol{M}^j\right)U^j = 0. \qquad (6)$$

Details of these matrices are described in ref[5].

3. Dispersion properties of guided waves propagating in helical structures

Fig. 2 shows dispersion curves for κ= 0 /m, τ= 0 /m and κ= 50 /m, τ= 50 /m obtained by the SAFE method. L(0,1)-mode, T(0,1)-mode and F(1,1)-mode change into L(0,1)-like-mode, T(0,1)-like-mode and two F(1,1) -like-modes, respectively, by the curvature and torsion. If the structure has a curvature, guided waves of L(0,1)-mode and T(0,1)-mode do not propagate in a lower frequency band.

Fig. 3 shows change of the phase velocity with the curvature at the input frequency of 0.01 MHz for a structure with τ= 0 /m and 50 /m. Fig. 4 shows the change of the phase velocity with the torsion at the input frequency of 0.01 MHz for a structure with κ= 0 /m and 50 /m. If the structure has a curvature, guided wave of F(1,1)-mode bifucates as the torsion increases.

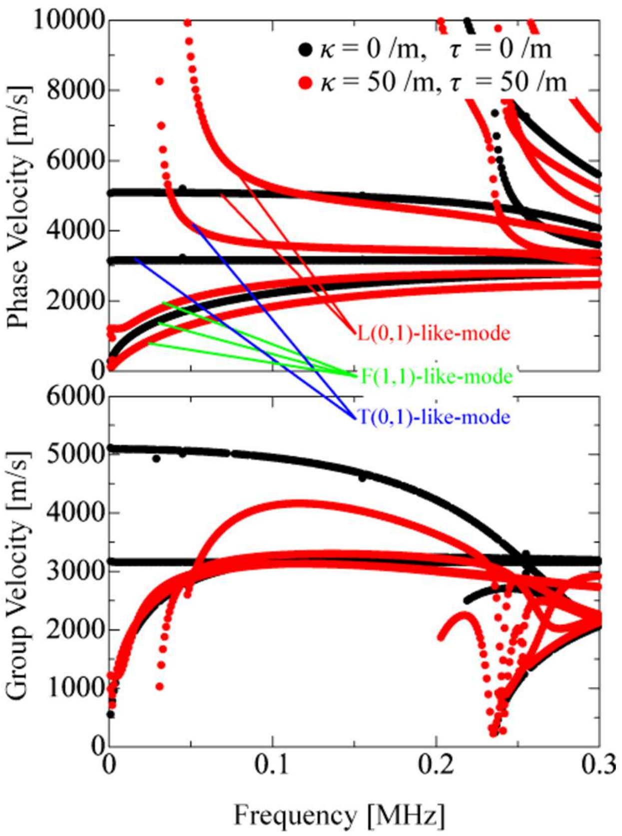

Fig. 2 Dispersion curves of helical structures

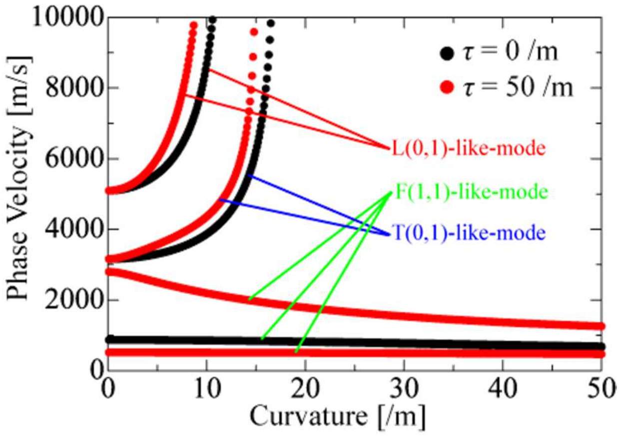

Fig. 3 Dependence of phase velocities on curvatures at the input frequency of 0.01 MHz

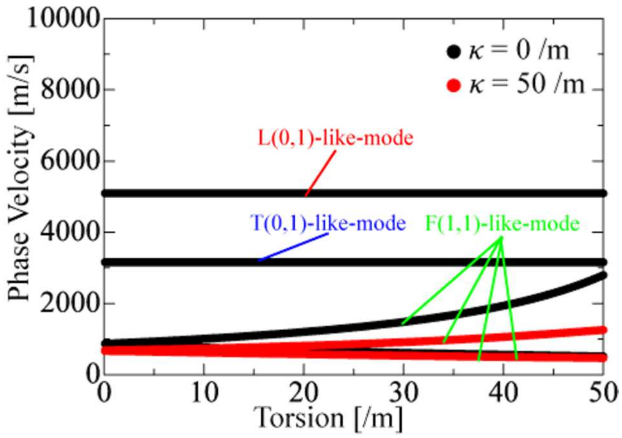

Fig. 4 Dependence of phase velocities on torsions at the input frequency of 0.01 MHz

4. Conclusions

A numerical method of calculating dispersion curves for helical structures has been developed by introducing the helical coordinate. The phase velocities of L(0,1)-mode and T(0,1)-mode increase as the curvature increases in a lower band.

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