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A Comparison Examination on Treatment of Interface between Different Media for Sound Field Simulation Using CIP Method

CIP-MOC 法による音場解析における媒質間境界の設定に関する検討

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1. Introduction

To date, as a result of advances in computer technology, numerical simulation for sound wave propagation has been frequently investigated. For sound field simulation, the development of accurate and effective analysis method in time domain has become an important technical issue.

The constrained interpolation profile(CIP) method, a novel low-dispersive numerical scheme, is a kind of method of characteristics(MOC) employing a co-located grid system. In our previous study, we have applied the CIP method to solving the linearized sound propagation equation [1-3].

In sound filed simulation using CIP method setting boundary interface between different media or material is required. That is, it is necessary to implement the reflection and transmission condition for sound fields and their spatial derivatives on a interface[3].

In this study, we implemented treatment of interface between different media in CIP sound field analyses and show the simulation result using CIP method. In adidtion, we compared the CIP method with Finite-difference time-domain(FDTD) method employing a staggered grid system.

2. Numerical Analysis Using CIP Method

Linearized governing equations of sound field are given in Eq. (1) and Eq. (2).

$$\rho\frac{\partial\vec{v}}{\partial t} = -\nabla\cdot p \qquad (1)$$

$$\nabla\cdot\vec{v} = -\frac{1}{K}\frac{\partial p}{\partial t} \qquad (2)$$

In these equations, ρ denotes the density of the medium, K is the bulk modulus, p is the sound pressure, and $\vec{v}$ is the particle velocity.

Here, assuming the analysis of sound field propagation of the x-direction, we can obtain the following equations from Eq. (1) and Eq. (2).

$$\frac{\partial}{\partial t}p + c\frac{\partial}{\partial x}Zv_x = 0 \qquad (3)$$

$$\frac{\partial}{\partial t}Zv_x + c\frac{\partial}{\partial x}p = 0 \qquad (4)$$

In these equations, Z indicates the characteristic impedance (i.e. $Z = \sqrt{K\rho}$) and c represents the sound velocity in the medium (i.e. $c = \sqrt{K/\rho}$).

Then, by addition and subtraction of Eq. (3) and Eq. (4), the advection equations are given as

$$\frac{\partial}{\partial t}(p \pm Zv_x) \pm c\frac{\partial}{\partial x}(p \pm Zv_x) = 0 \qquad (5)$$

In the CIP method, simple spatial differentiation of Eq. (5), the advection equations of their derivatives are also obtained as the following equations.

$$\frac{\partial}{\partial t}(\partial_x p \pm Z\partial_x v_x) \pm c\frac{\partial}{\partial x}(\partial_x p \pm Z\partial_x v_x) = 0. \qquad (6)$$

Therein $\partial_x = \partial x$.

We show the means to calculate the field at the n+1 time step from the field of n time step, applying the CIP method to discretized acoustic field components. The field components defined on grid points ($x = i\Delta x$) at the n time step are given as the following equations:

$$F^n_{x\pm}(i) = p^n(i) \pm Z(i \mp 1/2)v^n_x(i) \qquad (7)$$

$$G^n_{x\pm}(i) = \partial_x p^n(i) \pm Z(i \mp 1/2)\partial_x v^n_x(i) \qquad (8)$$

3. Treatment of interface between different media

Next, we present treatment of interface between different media. We assume that the interface between the medium 1 (ρ_1, K_1) and the medium 2 (ρ_2, K_2) is $x = i_0\Delta x$ and that their impedance are respectively

$$Z_1 = Z(i_0 - 1/2), \qquad (9)$$

$$Z_2 = Z(i_0 + 1/2). \qquad (10)$$

Then, from Eqs. (7) and (10), we can obtain the interface conditions of $F_{x\pm}$ and $G_{x\pm}$ as

$$F^n_{x2+}(i_0) = T_{12}F^n_{x1+}(i_0) + \Gamma_{21}F^n_{x2-}(i_0) \qquad (11)$$

$$F^n_{x1-}(i_0) = \Gamma_{12} F^n_{x1+}(i_0) + T_{21} F^n_{x2-}(i_0) \qquad (12)$$

$$G^n_{x2+}(i_0) = T'_{12} G^n_{x1+}(i_0) + \Gamma'_{21} G^n_{x2-}(i_0) \qquad (13)$$

$$G^n_{x1-}(i_0) = \Gamma'_{12} G^n_{x1+}(i_0) + T'_{21} G^n_{x2-}(i_0) \qquad (14)$$

In these equations,

$$T_{12} = \frac{2Z_2}{Z_1 + Z_2}, \qquad \Gamma_{12} = \frac{Z_2 - Z_1}{Z_1 + Z_2}, \qquad (15)$$

$$T_{21} = \frac{2Z_1}{Z_1 + Z_2}, \qquad \Gamma_{21} = \frac{Z_1 - Z_2}{Z_1 + Z_2}, \qquad (16)$$

$$T'_{12} = \frac{c_1}{c_2} T'_{12}, \qquad T'_{21} = \frac{c_1}{c_1} T'_{21}, \qquad (17)$$

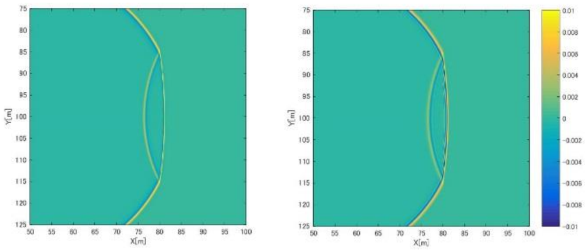
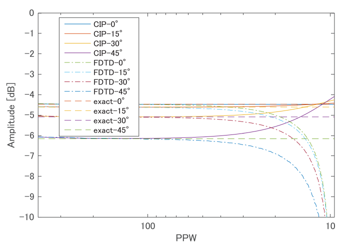


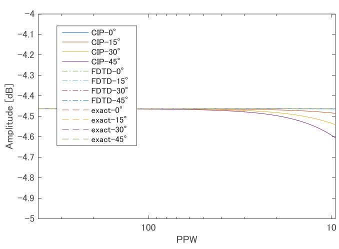
Fig. 1 Spatial distribution of sound pressure used in the CIP method (left) and the FDTD method (right).

Table 1 Value of the constant used in the analysis

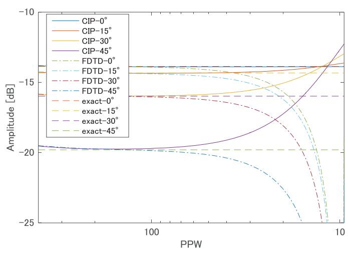
Δx	grid size (x-direction)	0.05[m]
Δy	grid size (y-direction)	0.05[m]
Δt	time step	50[μs]
NX	number of grid (x-direction)	3001
NY	number of grid (y-direction)	3001
c	Sound speed (Media 1)	343[m/s]
Z	The characteristic impedance (Media1)	415.03 [Pa*kg/m^3]



(a)



(b)



(c)

Fig. 2 Calculated results (a)Γ_{12} = 0.8 and c_2 = 1/3c_1. (b) Γ_{12} = 0.8 and $c_2 = c_1$. (c) Γ_{12} = 0.5 and c_2 = 1/3c_1.

$$\Gamma'_{12} = -\Gamma_{12}, \qquad \Gamma'_{12} = -\Gamma_{12}. \qquad (18)$$

Here, T and Γ are the transmission coefficients and reflection coefficients, respectively. In the CIP method, the interface conditions between different media are defined as above equations. Hence, respective procedure in the y-direction as well as in the x-direction can be applied.

4. Computational results

Calculation parameters used in the calculations are summarized in table 1. We compared simulation results using the CIP method and the FDTD method[4] by changing Z and c of Media 2.

Figure 1 shows the sound pressure distribution obtained by the CIP method and The FDTD method at n = 2000 time step. In this computation, we assume Γ_{12} = 0.5 (Z_2=3Z_1) and c_2 = 1/3c_1. We can ascertain the reflected waves from the interface in both figures.

Figures 2 shows the amplitude ratio of the input wave and reflected wave against points per wavelength (PPW) at each angle. Results by the CIP method, the FDTD method and theoretical value are depicted in these figures. In Fig. 2 (a), we set Γ_{12} = 0.8 (Z_2=9Z_1) and c_2 = 1/3c_1. We also assume that Γ_{12} = 0.8 (Z_2=9Z_1) and $c_2 = c_1$ in (b), and that Γ_{12} = 0.5 (Z_2=3Z_1) and c_2 = 1/3c_1 in (c).

From Fig. 2 (b), we can ascertain results of both the CIP method and FDTD method become different from the theoretical value as the angle is large. In Fig. 2 (a) and (c), it is clarified that the error of the CIP method is smaller than that of the FDTD method. On the other hand, we can see the difference between calculated results and theoretical value as reflection coefficient is larger.

5. Conclusion

We examined the treatment of interface between different media for sound field simulation using CIP method. We apply the conditions to 2-dimmensional computation of sound fields and evaluate each interface condition.

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