

Estimation of liquid properties by inverse problem analysis based on shear horizontal surface acoustic wave sensor responses

SH-SAW センサ応答を用いた逆問題解析による液体の物性値推定

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1. Introduce

A shear horizontal surface acoustic wave (SH-SAW) sensor can measure the liquid properties, such as viscosity, by detecting amplitude and phase changes due to liquid loading on the SH-SAW sensor. In our previous research, the perturbation equations were derived to explain the sensor responses⁽¹⁾. Also, the perturbation equations were compared with the numerical calculation results, which was based on the Campbell and Jones method⁽²⁾. In these analyses, the literature values of the liquid properties were used to obtain the amplitude and phase changes. This is the direct problem. For the actual application of the SH-SAW sensor, the liquid properties shall be obtained from the measured amplitude and phase. This is the inverse problem.

In this paper, we developed the inverse problem analysis method to estimate the liquid properties (density and viscosity) from the SH-SAW sensor responses. Glycerol or ethanol solution was measured by using the SH-SAW sensor. Then the estimated results were compared with the literature values.

2. SH-SAW sensor

The SH-SAW sensor used was fabricated on 36°YX-LiTaO₃ (see **Figure 1**). The center frequency of it was 51.5 MHz. Liquid pool was mounted onto the propagation surface. Using the vector voltmeter, amplitude and phase changes were monitored. The attenuation change normalized by the wave number, $\Delta\alpha/k$, and velocity change, $\Delta V/V$, were derived from amplitude and phase changes, respectively.

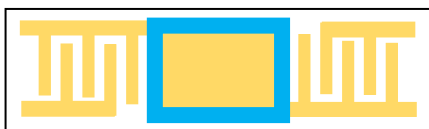


Fig. 1 The SH-SAW sensor with liquid pool.

3. Direct problem analysis

Moriizumi, et al. expanded the numerical calculation method for liquid/piezoelectric substrate structure⁽³⁾. The liquid layer model was adapted by the Hayasaka's method⁽⁴⁾. The liquid is characterized by the bulk modulus, density, and viscosity. If material constants and Euler angle of the piezoelectric substrate is known, the $\Delta\alpha/k$ and $\Delta V/V$ are obtained for a liquid using the direct problem analysis.

4. Inverse problem analysis

As the perturbation method is one of the approximation, it is difficult to obtain the precise values of the liquid properties. Therefore, in this paper, the inverse problem analysis is proposed to determine those. In this paper, the bulk modulus of the liquid was fixed as water, and the density and viscosity were varied. Steps of the inverse problem analysis are shown in the following.

Step(1). Using the direct problem method, $\Delta\alpha/k$ and $\Delta V/V$ are calculated by using arbitrary values of the density, ρ , and viscosity, η .

Step(2). Density and viscosity were varied and $\Delta\alpha/k$ and $\Delta V/V$ were obtained. **Figure 2** shows $\Delta\alpha/k$ map as functions of density and viscosity.

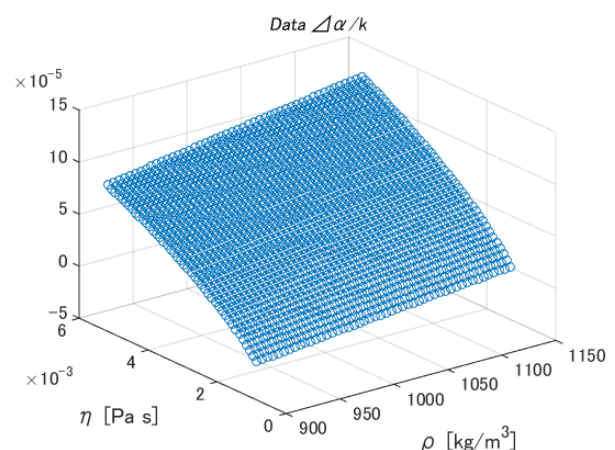


Fig. 2 $\Delta\alpha/k$ map.

Step(3). Derive the approximate functions of $\Delta\alpha/k(\rho, \eta)$ or $\Delta V/V(\rho, \eta)$ from the maps by using a nonlinear least-squares method. Especially, a Marquardt method⁽⁵⁾ was selected in this study. The method has the advantage in terms of improvement of convergence speed and suppression of divergence. Approximate functions are defined as follows.

$$\Delta\alpha/k(\rho, \eta) = a(1)\log\rho + a(2)\log\eta + a(3)\rho + a(4)\eta + a(5)\rho\eta + a(6) \quad \dots(1)$$

$$\Delta V/V(\rho, \eta) = b(1)\log\rho + b(2)\log\eta + b(3)\rho + b(4)\eta + b(5)\rho\eta + b(6) \quad \dots(2)$$

Here, a and b are coefficient and derived in this step.

Step(4). Substitute $\Delta\alpha/k$ and $\Delta V/V$ obtained from the measurement on the left side of eqs. (1) and (2).

Step(5). Calculate the simultaneous equation of eqs. (1) and (2) by Newton's method and derive the estimated values of the density and viscosity.

5. Comparison of estimated values and literature values

Table. I shows the comparison results of estimated and literature values with the glycerol solution (5, 10, 30, and 50 wt%). Literature values were referred to ref. (6). From results, when the low concentration, the estimated and literature values agree well. However, the differences increase with increasing concentration. In this paper, the bulk modulus was fixed. This assumption is not satisfied at high concentration glycerol solution. Also, when the concentration of glycerol increase, the viscoelastic effect must be considered⁽⁷⁾. For this reason, it cannot be applied calculated analysis value for the Newtonian fluid.

The results for the ethanol solution is shown in **Table. II**. Whereas the density is accurately estimated, the viscosity values are not estimated. However, the ethanol has a small elastic modulus as compared with glycerol, so that the difference between estimated and literature values is small. Therefore, it is possible to estimate of liquid properties.

6. Conclusion

In this paper, the estimation method of the liquid properties is proposed from the SH-SAW sensor response. Glycerol or ethanol solution was used as the sample liquid. We compared the estimated values derived from the inverse problem analysis proposed with the literature values. For the glycerol solution, those values do not agree.

Table. I Comparison results of glycerol solutions.

Density [kg/m ³]				
	5wt%	10wt%	30wt%	50wt%
Literature	1.009E+03	1.021E+03	1.071E+03	1.124E+03
Estimated	9.985E+02	1.204E+03	1.603E+03	2.081E+03
Difference	-1.04%	+18.00%	+49.72%	+85.17%

Viscosity [Pa s]				
	5wt%	10wt%	30wt%	50wt%
Literature	1.036E-03	1.170E-03	2.185E-03	5.105E-03
Estimated	1.090E-03	9.237E-04	1.304E-03	2.706E-03
Difference	+5.21%	-21.50%	-40.34%	-47.00%

Table. II Comparison results of ethanol solutions.

Density [kg/m ³]				
	5wt%	10wt%	30wt%	50wt%
Literature	9.893E+02	9.819E+02	9.539E+02	9.139E+02
Estimated	9.986E+02	9.687E+02	9.740E+02	9.599E+02
Difference	+0.94%	-1.34%	+2.11%	+5.03%

Viscosity [Pa s]				
	5wt%	10wt%	30wt%	50wt%
Literature	1.228E-03	1.442E-03	2.667E-03	2.813E-03
Estimated	1.018E-03	1.419E-03	3.035E-03	3.154E-03
Difference	-17.14%	-1.58%	+13.79%	+12.11%

On the other hand, for the ethanol solution, the liquid properties can be estimated. Therefore, the proposed method is effective when the bulk modulus assumes as constant.

Acknowledgment

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